Free surface simulation of a two-layer fluid by boundary element method

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ABSTRACT: *A two-layer fluid with free surface is simulated in the time domain by a two-dimensional potential-based Numerical Wave Tank (NWT). The developed NWT is based on the boundary element method and a leap-frog time integration scheme. A whole domain scheme including interaction terms between two layers is applied to solve the boundary integral equation. The time histories of surface elevations on both fluid layers in the respective wave modes are verified with analytic results. The amplitude ratios of upper to lower elevation for various density ratios and water depths are also compared.*

KEY WORDS: Two layer fluid; Internal wave; Time domain; Numerical wave tank; Leap frog integration.

INTRODUCTION

In the classical view of hydrodynamics calculation, water density is assumed to be uniform and the results came from this premise have been satisfied with most of physical phenomena in the ocean. In some special areas, however, water density is not uniform and two-layer density fluids may exist, caused by changing of the water temperature and salinity. For instance, when warm sea water meets cold water with high salinity, two-density layers can be formed and the internal waves may be generated at the interface between the layers.

The restoring force for water waves, called relative buoyancy force, is proportional to the product of gravity and the density difference between two layers. At the interface between two layers the difference is much smaller than the density difference between air and water. Consequently, internal waves generated at the interface can attain much larger amplitudes than free surface waves. It also takes longer for the restoring force to return water particles to their average positions, and the wave has a period much longer than surface gravity waves (e.g., from 10 minutes to several hours).

The internal waves are considered as a cause of damage of ships and offshore structures due to their huge magnitudes. Several studies related to these unique waves were conducted after the first observation by Russell (1838, 1844). Osborne et al. (1978) researched the influence of internal waves on deep sea drilling. Linton and McIver (1995) studied the horizontal cylinders interacted with waves in two-layer fluids. Kuznetsov et al. (2003) also studied wave interaction with a 2D floating body in a two-layer fluid. Recently, the influence of local shear flow on the cross flow response of a cylindrical structure was reported by Kim and Rheem (2009).

When two-layer fluids are considered as a two-domain problem, two wave modes are obtained such as surface wave mode (S-mode) and internal wave mode (I-mode). The propagation of internal waves can be predicted by the modified dispersion relation of respective wave mode. Thus, the waves on both fluid surfaces propagate with two different wave numbers k_s and k_i corresponding to S-mode and I-mode, respectively. The wave number of S-mode (k_s) is smaller than that of I-mode (k_i); thus the propagating waves in S-mode are longer than those in I-mode. The generated waves on free surface and interface in S-mode are in phase and the elevation of free surface is bigger than that of interface. However, in I-mode, the waves are 180 degrees out-of-phase and the free surface amplitudes are usually smaller than the internal waves.

For wave-body interaction in two-layer fluids, Yeung and Nguyen (1999) developed the boundary integral equation method to solve the radiation and diffraction problem for a rectangular barge with finite depth in the frequency domain. Ten and Kashiwagi (2004), Kashiwagi et al. (2006) and Kim and Koo (2010) also studied for a 2D floating body in two-layer fluids with a similar manner.

Even if numerous computational studies of internal waves have been conducted so far, most of calculations are based on frequency domain, assuming of small harmonic amplitudes. However, the internal waves have large amplitudes and long periods, and their interaction with structure may be highly nonlinear. Therefore, a time domain approach has to be required to investigate the nonlinear characteristics of internal waves and their interaction with structures.

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In the present study, a 2-dimensional Numerical Wave Tank (NWT) technique based on a two-domain Boundary Element Method in the time domain is developed to simulate internal waves in two-layer fluids. The computational domain is assumed to be filled with potential fluids of two different densities and a flat sea bottom.

The boundary integral equation transformed from the governing equation of both fluid domains is solved by using the whole domain scheme to satisfy the interface condition of two computational domains. The leap-frog time marching scheme is newly developed for time integration of surface boundary conditions on both domains. The time-varying waves on both surfaces are measured to investigate the characteristics of internal waves for various ratios of fluid density and water depth.

The present simulation is based on the leap-frog time integration scheme and this time-domain method can be easily extended to fully nonlinear time simulation as changing the instantaneous surface boundaries with updating the node locations. Therefore, the present work is a worthwhile study for solving the fully nonlinear time domain solution of two-layer fluid in the next study.

Since the stratification of two-density fluids is assumed to be stable, the present calculation condition is based that no fluid-mixing occurs at the interface during the time simulation.

MATHEMATICAL FORMULATION

Governing Equation and Boundary Integral Equation

The computational domain composed of two density-fluids is assumed to be filled with inviscid, incompressible and irrotational fluids. Assuming that the water particle has a harmonic motion of angular frequency ω, the velocity potential is defined as Eq. (1):

$$\phi(\vec{x}, t) = \text{Re}[\Phi(\vec{x})e^{-i\omega t}] \tag{1}$$

The gradient of the velocity potential can express water particle velocity, and the continuous equation is satisfied in the computational domain, thus the Laplace equation can be used as a governing equation.

$$\nabla^2 \phi^{(m)} = 0 \tag{2}$$

where m=1, 2 denote upper and lower fluid domains, respectively and surface elevation on each boundary is also given as:

$$\eta^{(m)}(x, t) = a_0^{(m)} \cos(kx - \omega t) \tag{3}$$

where $a_0^{(m)}$ is wave amplitude on respective fluid surfaces.

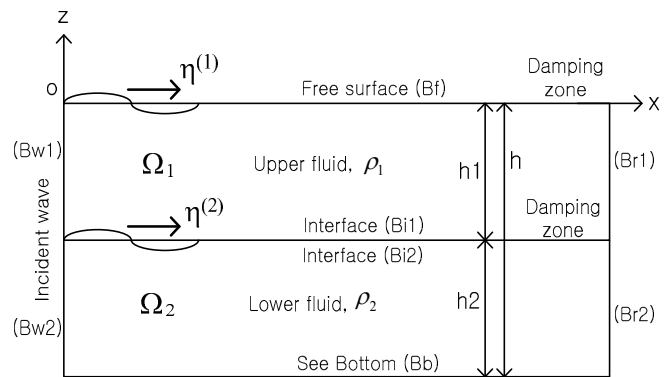Fig. 1 shows the entire computational domain with symbols of parameters.

Fig. 1 Computational domain of two layered fluids.

Using the Green function satisfying the Laplace equation, the governing equation can be transformed to a boundary integral equation (Eq. (4)).

$$\alpha \phi_i^{(m)} = \iint\limits_{\Omega_{(m)}} (G_{ij}^{(m)} \frac{\partial \phi_j^{(m)}}{\partial n} - \phi_j^{(m)} \frac{\partial G_{ij}^{(m)}}{\partial n}) ds \tag{4}$$

where the Green function is $G_{ij}(x_i, z_i, x_j, z_j) = -(1/2\pi) \ln R_1$ for a two-dimensional problem in each domain (Ω_1, Ω_2), α is a solid angle (it is 0.5 when singularities are on the boundary) and R_1 is the distance between source (x_i, z_i) and field points (x_j, z_j). It is noted that the source and field points to obtain the Green function in the present study were defined only in the same fluid domain, i.e. the Green function for the upper domain ($G_{ij}^{(1)}$) is obtained from the collocation points only in the upper domain.

Boundary Conditions

In order to solve the boundary integral equation the corresponding boundary conditions for both fluid domains should be defined which include free surface, interface, rigid end-wall, sea bottom, and incident wave boundaries.

Linearized dynamic and kinematic free surface boundary conditions are described as:

$$\frac{\partial \phi}{\partial t} = -g\eta^{(1)} - \frac{P}{\rho} \qquad \text{on} \quad z = 0 \tag{5}$$

$$\frac{\partial \eta^{(1)}}{\partial t} = \frac{\partial \phi^{(1)}}{\partial z} \qquad \text{on} \quad z = 0 \tag{6}$$

where g is gravity and air pressure (P) on the free surface is set to be zero.

The linearized dynamic condition on the interface is related to the density ratio of upper to lower fluids, and the water particle velocity on the interface is continuous, thus the dynamic and kinematic interface boundary conditions are given as: