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Finite element method for one-dimensional rill erosion simulation on a curved slope

Lijuan Yan[a], Tingwu Lei[b,*], Jing Zhang[b], Qingwen Zhang[c], Liqin Qu[d]

[a]Bureau of Comprehensive Development, Ministry of Water Resources, Beijing 100053, PR China

[b]Key Laboratory of Modern Precision Agriculture Integration Research, College of Hydraulic and Civil Engineering, China Agricultural University, Beijing 100083, PR China

[c]Institute of Environment and Sustainable Development in Agriculture, CAAS, Beijing 100081, PR China

[d]International Research and Training Center on Erosion and Sedimentation, Beijing 100048, PR China

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Abstract

Rill erosion models are important to hillslope soil erosion prediction and to land use planning. The development of rill erosion models and their use has become increasingly of great concern. The purpose of this research was to develop mathematic models with computer simulation procedures to simulate and predict rill erosion. The finite element method is known as an efficient tool in many other applications than in rill soil erosion. In this study, the hydrodynamic and sediment continuity model equations for a rill erosion system were solved by the Galerkin finite element method and Visual C++ procedures. The simulated results are compared with the data for spatially and temporally measured processes for rill erosion under different conditions. The results indicate that the one-dimensional linear finite element method produced excellent predictions of rill erosion processes. Therefore, this study supplies a tool for further development of a dynamic soil erosion prediction model.

Keywords: Finite element method; Simulation; Rill erosion; Dynamics; Galerkin

1. Introduction

Soil eroded by shallow water flows such as over land or in-rill flow is a critical component of the erosion system on upland areas. The erosion processes of sediment detachment, transport, and deposition on hillslope areas are complex and interactive. This has been of great interest to mathematical and computer modelers for upland erosion modeling. A number of process-based dynamic models have been developed to better understand and model the processes of runoff and soil erosion under different situations. The water erosion prediction project (WEPP) model (Ascough, Baffaut, Nearing, & Liu, 1997; Flanagan & Nearing, 1995), European soil erosion (EuroSEM) model (Morgan, Quinton, & Rickson, 1992), the areal nonpoints source watershed environment response simulation (ANSWERS) model (Beasley, Huggins, & Monke, 1980), the Limburg soil erosion model (LISEM) (De Roo,

*Correspondence to: China Agricultural University, Qinghua Donglu, Beijing 100083, PR China. Fax: +86 10 6273 6367.

E-mail address: leitingwu@cau.edu.cn (T. Lei).

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Offermans, & Cremers, 1996), Griffith University Erosion System Template (GUEST) (Misra & Rose, 1996) are but some examples. However, most of the models are either empirical or using Finite Difference Method (FDM) to solve the mathematic equations. Model performances are either not efficient or are difficult to deal with complex boundary conditions as associated with FDM. Furthermore, model parameters are often, if not impossible, difficult to directly measure to present the actual physical properties (Lane, Shirley, & Singh, 1988).

Generally, soil erosion occurs on a three-dimensional (3-D) surface. But soil erosion behavior in a rill could to some extent be well described with a one-dimensional (1-D) model. And the dynamic process of soil erosion along a streamline of overland flow could be approximated with a one-dimensional model for the purpose of understanding the basic mechanics behind the complicated phenomenon. The governing equations are a set of partial differential equations involved in the hydrologic erosion processes. Numerical techniques do not need to make as many assumptions as required for analytic solutions. The driving force that is the rainfall excess term can vary with time and space (Lane et al., 1988; Sharda, Singh, Sastry, & Dhruvanarayana, 1994). Numerical methods such as the finite difference method (FDM) and the finite element method (FEM) are employed to solve these equations. Finite element techniques are generally recognized to have significant advantages over finite difference procedures for irregularly shaped flow regions (Zienkiewics, Taylor, & Zhu, 2013) FEM is now widely used to solve a variety of important problems in the field of soil science and groundwater hydrology. A lot of works has been accomplished in early studies (Bralts & Segerlind, 1985; Guymon, 1972; Jayawardena & White, 1977; Ross, Contractor, & Shanholtz, 1979; Taylor, Al-Mashidani, & Davis, 1974). Recently, Celia, Bouloutas, and Zarba (1990) stated that numerical approximations based on different forms of the governing partial differential equation can lead to significantly different results for unsaturated flow problems. A finite element model simulating runoff and soil erosion from agricultural lands has been developed by Sharda and Nearing (1999). The sediment continuity equation was solved employing a fully implicit scheme for time integration. The complete Yalin's equation (Yalin, 1977) was used to simulate sediment transport capacity (Sharda et al., 1994; Sharda & Nearing, 1999; Sharda & Singh, 1994). Jaber and Mohtar (2002) evaluate the stability and accuracy of finite element schemes for the one-dimensional kinematic wave solution. They believed the lumped scheme considerably reduces oscillations without significant reduction in the overall solution accuracy. A kinematic wave based distributed watershed model using the finite element method, GIS and remotely sensed data has been reported by Venkata, Eldho, Rao, and Chithra (2008). This model could simulate hydrographs reasonably well at the outlet of the watershed.

The purposes of this study are: to develop the mathematic models for one-dimensional rill soil erosion on hillslope, including the hydrodynamics of shallow water flow along rills on the curved slopes, the soil detachment/deposition and transportation; to develop numerical algorithms and the FEM formulations for simulating the spatial and temporal processes; and, to validate the procedures by comparing the computed results with laboratory experimental data.

2. The mathematical models

Basic assumptions for deriving the mathematic model for soil erosion from 1-D rill under the impact of water flow are: (1) the water is very shallow, compared with the length; (2) velocity in the vertical direction is a uniformly distributed flow profile and (3) Velocity is that of depth-averaged. Based on those assumptions the mathematic models for rill erosion are given as the following.

2.1. Hydrodynamic models

The derivations of the continuity and momentum equations for overland and channel flow can be found in a number of references (Ascough et al., 1997; Kibler & Woolhiser, 1970; Lei, Nearing, Haghighi, & Bralts, 1998; Sharda & Nearing, 1999; Tsai & Yang, 2005). For mass conservation, the governing equation is written as

$$\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} = \sigma \tag{1}$$