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Low Reynolds number drag and particle collision efficiency of a cylindrical fiber within a parallel array

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ABSTRACT

Accurate numerical methods were used for a systematic study of the drag coefficient and the particle collision efficiency of cylindrical filter fibers over a wide range of non-zero Reynolds numbers Re and inter-fiber distances s/d_F of practical importance to gas filtration applications.

On the basis of the numerical flow field data, a novel fit function was derived for the *fiber drag coefficient* c_D as a function of s/d_F and Re for the parameter range $2 \le s/d_F \le 20$ and $Re \le 20$.

In the second part of the paper CFD and Miyagi-type flow fields were used to calculate the *collision efficiency* of spherical, non-diffusive particles for Stokes numbers of $0 \le St \le 10^3$ and interception parameters of $0.005 \le R \le 0.5$. From these numerical data, novel fit functions were derived for the collision efficiency by interception only, and for the combined efficiency by inertia and interception. The effects of fiber spacing s/d_F and Re on particle inertia are accounted for by a modified Stokes number.

1. Introduction

Fibrous filters are widely used to remove particles from gases due to their great versatility and cost-effectiveness. On the other hand, the diversity of performance requirements for filters in terms of efficiency, pressure drop, dust holding capacity, operating conditions, geometrical configuration and materials of manufacture has also become extremely broad. Manufacturers therefore resort increasingly to software simulation tools to optimize filters for a specific application.

Most simulation tools are ultimately based on numerically reliable *fiber drag* and *single fiber efficiency* models, unless they compute filter behavior from first principles. Unfortunately, fiber efficiency and drag are very sensitive to parameters such as particle size, inter-fiber spacing and flow conditions, which makes it difficult to formulate quantitatively accurate models capable of covering the entire range of operating conditions required for simulation. This is aggravated by the often overlooked fact that most filter media have a rather non-uniform internal structure resulting in large variations of local flow velocity. In other words, while the mean flow velocity may still be well within the range of validity of an expression, the local conditions inside the matrix may vary by an order of magnitude.

Despite decades of research and copious, mostly well-known literature, the existing body of models for fiber drag and efficiency (reviewed in the following chapters) still represents a kind of patchwork with non-overlapping boundaries of

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validity and other practical problems, which are usually not obvious until one begins to compute and compare models. The lack of broadly applicable models is particularly apparent in the *regime dominated by particle inertia (in combination with fiber interception) at low and moderate Re numbers*, where no suitable analytical flow field formulations (such as Kuwabara, 1959) are available. The inertial regime is important however for industrial applications, where filter performance tests are often based on fairly coarse dust.

Why does the current paper focus on an array of parallel equidistant fibers? This choice of model geometry is based on past experience with the simulation of particle capture efficiencies and dust distributions inside gas filter media (Hoferer, 2011), which showed that single-fiber efficiencies obtained from arrays gave a more realistic agreement with the behavior of real media, than efficiencies of isolated fibers (Kasper et al., 2009). These simulations also showed that reliable analytical expressions for capture efficiency and drag were lacking even for bare fibers, especially in a range of Reynolds numbers $0 < Re \le 5$.

The objective of this work was to fill this gap with new expressions with a physically meaningful behavior. The approach taken was to first perform accurate numerical simulations of drag and particle capture efficiency over a wide range of parameters. Analytical fit functions were then developed on the basis of new dimensionless quantities, which “collapse” these numerical data into analytical expressions.

2. Existing models for flow field and drag on a cylinder within an array as a function of Reynolds number and fiber spacing

In the following, we shall always consider a vertically downward flow of gas (i.e. in negative *y*-direction) through an infinite array of horizontal, equidistant, identical, parallel cylinders of diameter d_F as shown in Fig. 1. The undisturbed gas velocity is called v_∞. The fiber offset s is the center-to-center distance of two adjacent fibers.

The flow Reynolds number is defined as usual, on the basis of gas density ρ_G and dynamic viscosity μ:

$$Re = \frac{\rho_G d_F v_\infty}{\mu}$$

The drag coefficient c_D is a non-dimensionalized drag force F_D acting on a straight cylinder of length L_F as defined via the expressions

$$F_D = c_D \frac{\rho_G}{2} v_\infty^2 \; d_F \; L_F = \frac{1}{2} c_D \; Re \; \mu \; v_\infty \; L_F$$

2.1. Miyagi flow field for **Re**→0

Based on a model proposed by Tamada & Fujikawa (1957) for low *Re*, Miyagi (1958) developed an analytical approximation to the velocity field around a cylinder within an infinite parallel array for the case $Re \to 0$. Therein, the fiber represents a disturbance v_S^* imposed on the normalized uniform flow $\overrightarrow{v^*} = (0, -1)$, which is expressed as a function of the dimensionless fiber offset s/d_F:

$$v_S^* = a_0\left[\ \ln(2\sinh\bar{\zeta}) + (\ln 2\sinh\zeta) - (\zeta+\bar{\zeta})\coth\zeta\right] \\ + \sum_{n=1}^{\infty} a_{2n}\left[\frac{d^{2n-1}}{d\bar{\zeta}^{2n-1}}\coth\bar{\zeta} - (\zeta+\bar{\zeta})\frac{d^{2n}}{d\zeta^{2n}}\coth\zeta\right] + \sum_{n=1}^{\infty} b_{2n}\frac{d^{2n-1}}{d\zeta^{2n-1}}\coth\zeta$$

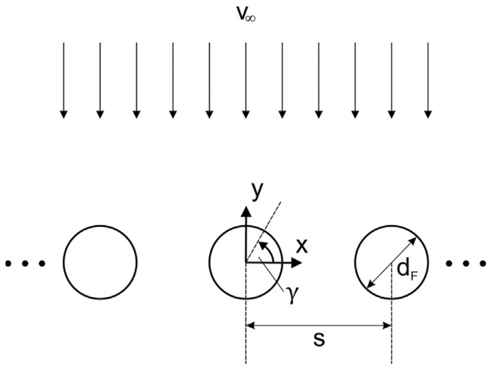

Fig. 1. Basic geometry setup.