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Effect of an external electric field on the charge distribution of electrostatic coagulation

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Abstract

This article analyzes the effect of an external electric field on the charge distribution of bipolar and unipolar charged particles numerically by solving the coagulation equation for charged particles, based on the analytical expression for the coagulation coefficient [Wang, L. Z., Zhang, X. R., & Zhu, K. Q. (2005). An analytical expression for the coagulation coefficient of bipolarly charged particles by an external electric field with the effect of Coulomb force. *Journal of Aerosol Science*, 36, 1050–1055]. For symmetric bipolar charged particles, the external electric field does not change the symmetry of the initial charge distribution as the coagulation time increases. In addition, the particle number concentration decays monotonically during coagulation regardless of the magnitude of the particle charge. For asymmetric bipolar charged and unipolar charged particles, however, the particle number concentration does not decay monotonically for each value of charge during coagulation, due to the effect of an external electric field.

Keywords: Charge distribution; External electric field; Electrostatic dispersion

1. Introduction

In order to analyze the dynamics of the aerosol charge distribution of charged particles based on the coagulation equation for charged particles, the coagulation coefficient of charged particles should first be obtained analytically (Eliasson & Egli, 1991; Oron & Seinfeld, 1989; Vemury, Janzen, & Pratsinis, 1997). Since the analytical expression for charged particles subject to a Coulomb force with and without the effect of an external electric field has been obtained (Wang, Zhang, & Zhu, 2005), it is convenient to analyze the corresponding dynamics of the aerosol charge distribution. Here we solve the coagulation equation, using the sectional method for computation of the size-charge distribution (Gelbard, Tambour, & Seinfeld, 1980; Oron & Seinfeld, 1989), with the coagulation coefficient of charged particles including the effect of an external electric field.

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2. Theoretical analysis

The basic conservation equation for the time evolution of the size-charge distribution of electrically charged particles is given by

$$\frac{\partial n(x,\phi,t)}{\partial t} = \frac{1}{2}\sum_{\phi'=-\infty}^{\infty}\int_0^x K_{\phi',\phi-\phi'}(x',x-x')n(x',\phi',t)n(x-x',\phi-\phi',t)\,\mathrm{d}x' - n(x,\phi,t)\sum_{\phi'=-\infty}^{\infty}\int_0^{\infty} K_{\phi\phi'}(x,x')n(x',\phi',t)\,\mathrm{d}x' - \frac{B(x)e^2\phi n\,(x,\phi,t)}{\varepsilon_0}\times\sum_{\phi'=-\infty}^{\infty}\phi'\int_0^{\infty} n(x,\phi',t)\,\mathrm{d}x, \tag{1}$$

where $n(x,\phi,t)\,\mathrm{d}x$ is the total number concentration of particles with volumes lying between x and $x+\mathrm{d}x$ and electrostatic charge ϕe at time t, e is the elementary unit of charge, ε_0 is the permittivity of free space, $B(x)$ is the particle mobility given by

$$B(x) = \frac{C_\mathrm{c}}{3\pi\mu d_\mathrm{p}}, \tag{2}$$

where C_c is the Cunningham correction factor (Hinds, 1999), μ is the viscosity of the air, and d_p is the particle diameter. $K_{\phi\phi'}(x,x')$ is the coagulation coefficient for two particles with volumes x, x' and charges ϕe, $\phi' e$, respectively. The analytical expression for the coagulation coefficient of charged particles with the effect of an external electric field is given by Wang et al. (2005)

$$K_{\phi\phi'}(x,x') = \frac{2k_\mathrm{B}T}{3\mu}\left(2+\frac{d_\mathrm{p}(x)}{d_\mathrm{p}(x')}+\frac{d_\mathrm{p}(x')}{d_\mathrm{p}(x)}\right)f_{\phi\phi'} + \frac{(d_\mathrm{p}(x)+d_\mathrm{p}(x'))^2}{12\mu}\left|E_0\left(\frac{\phi e}{d_\mathrm{p}(x)}C_\mathrm{c}(x)-\frac{\phi' e}{d_\mathrm{p}(x')}C_\mathrm{c}(x')\right)\right|, \tag{3}$$

$$f_{\phi\phi'} = \frac{\alpha_{\phi\phi'}}{\exp(\alpha_{\phi\phi'})-1}, \tag{4}$$

$$\alpha_{\phi\phi'} = \frac{\phi\phi' e^2}{2\pi\varepsilon_0(d_\mathrm{p}(x)+d_\mathrm{p}(x'))k_\mathrm{B}T}, \tag{5}$$

where $d_\mathrm{p}(x)$, $d_\mathrm{p}(x')$ are the particle diameters with volumes x, x', and $C_\mathrm{c}(x)$, $C_\mathrm{c}(x')$ are the Cunningham correction factor for particles with volume x, x', E_0 is the external electric field, k_B is the Boltzmann constant, and T is the absolute temperature.

The first term on the right-hand side of Eq. (3) accounts for the coagulation coefficient of charged particles subject to the Coulomb force without the effect of an external electric field (Fuchs, 1964; Zebel, 1958). The second term expresses the coagulation coefficient of charged particles subject to an external electric field without the effect of the Coulomb force (Wang et al., 2005). The coagulation coefficient of charged particles subject to the Coulomb force including the effect of an external electric field is assumed to consist of two terms above. This assumption is based on the fact that the Coulomb force is strong at short distances while the applied external electric field accelerates the coagulation at long distances. Consequently, the two effects work independently each other.

The first term on the right-hand side of Eq. (1) expresses the production rate of particles with volume x and charge ϕe due to coagulation between particles having volumes x', $x-x'$ and charges $\phi' e$, $(\phi-\phi')e$. The second term expresses the loss rate of particles with volume x and charge ϕe due to their coagulation with any other particle. The third term expresses a change in particle concentrations due to electrostatic dispersion.

Analytical solutions of Eq. (1) do not exist. To solve Eq. (1) numerically, the sectional method was used (Fujimoto, Kuga, Pratsinis, & Okuyama, 2003; Gelbard et al., 1980; Oron & Seinfeld, 1989). The basic idea of the sectional method