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Performance analysis based on least squares and extended Kalman filter for localization of static target in wireless sensor networks ☆

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ABSTRACT

Wireless sensor network localization is an essential problem that has attracted increasing attention due to wide demands such as in-door navigation, autonomous vehicle, intrusion detection, and so on. With the *a priori* knowledge of the positions of sensor nodes and their measurements to targets in the wireless sensor networks (WSNs), i.e. *posterior* knowledge, such as distance and angle measurements, it is possible to estimate the position of targets through different algorithms. In this contribution, two commonly-used approaches based on least-squares and Kalman filter are described and analyzed for localization of one static target in the WSNs with distance, angle, or both distance and angle measurements, respectively. Noting that the measurements of these sensors are generally noisy of certain degree, it is crucial and interesting to analyze how the accuracy of localization is affected by the sensor errors and the sensor network, which may help to provide guideline on choosing the specification of sensors and designing the sensor network. In addition, the problem of optimal sensor placement is also addressed to minimize the localization error. To this end, theoretical analysis have been made for the different methods based on three typical types of measurement noise: bounded noise, uniformly distributed noise, and Gaussian white noise. Simulation results illustrate the performance comparison of these different methods, the theoretical analysis and simulations and the optimal sensor geometry which may be meaningful and guideful in practice.

1. Introduction

Wireless sensor networks (WSNs) have attracted worldwide attention with the recent advances in micro-electro-mechanical systems (MEMS) technology, wireless communications, and digital electronics. A wireless sensor network consists of lots of low-cost, low-power, multi-functional sensors nodes with sensing, data processing, and communicating components, which are densely deployed to monitor the physical environment cooperatively [1,2]. WSNs have great potentials for many applications in scenarios such as autonomous vehicles [3], battlefield monitoring [4], target tracking and surveillance [5], intrusion detection [6], natural disaster relief [7], biomedical health monitoring [8], volcano monitoring [9], and seismic sensing [10]. Target localization is one of the most fundamental tasks for WSNs [11]. As facing cost,

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power, and other constraints, precise and low-cost localization is a critical requirement in WSNs.

Many researchers have focused on the problem of localization [12–15]. According to [16], localization techniques can be divided into two categories based on the communication among the sensor nodes: centralized localization and decentralized localization techniques [17]. For different mechanisms, we can also divide these methods into range-based and range-free schemes. The former needs to measure the distance or angle for position estimating based on time-of-arrival (TOA) [18], time-difference-of-arrival (TDOA) [19], received signal strength (RSS) [20], or angle-of-arrival (AOA) [21]. For range-free approaches, it is unavailable to get these measurements and they rely mostly on such information as proximity, i.e. neighbor connectivity, or hop-count in multi-hop WSNs [22,23]. In this paper, we merely discuss the situation of range-based schemes.

For localization problem in WSNs, algorithm design and error analysis are of great importance in wireless sensor networks. Since we aim at precise localization, what we concentrate on is not only how to locate the target, but also the accuracy degree or the error bound of the localization result. With these considerations, one interesting and crucial problem is—*Can we obtain better localization performance according to different kinds of measurements or other realistic cases through algorithm selection and sensor network design?* Obviously the answer to this problem is not trivial and detailed investigations to this problem may bring through direct benefits to applications of WSNs. Note that in practice, considering the demands of lower complexity and real-time computing, it may not be necessary to take all possible algorithms and situations in consideration, which may result in unnecessary complexity in computation or implementation.

Several methods were described in [24] through different measurements such as a distance and a direction, two directions, or three distances. Least-squares (LS) was widely used for position estimation [25–30]. A new method was presented by splitting the complex least-squares algorithm into a less central precalculation and a simple, distributed subcalculation in [31]. For underwater wireless sensor networks (UWSN), a novel least-squares method based on energy measurements was proposed in [32]. In [33], Born and Reichenbach presented a technique to convert the complex nonlinear Least Squares calculation and distribute the tasks over the network effectively. Kalman filter can be also used for localization in WSNs [17,34]. Rao and Durrant-Whyte [17] presented a fully decentralized Kalman filter algorithm which ensured the ideal implementation on a parallel processing array. In [34], RF mapping and Kalman filter were used to initialize the position and update the estimation using the distance measurements respectively.

For error analysis, Zhang et al. [35] analyzed some possible conditions for unique localization based on distance or bearing constraints and their combination respectively. Crámer–Rao lower bound (CRLB) is also widely used which is an algorithm-independent method [36–40]. In [36], the CRLB was given under anchored localization and anchor-free localization. Literature [38] dealt with the localization errors in distance-based one-dimensional sensor networks. The fundamental behaviors of localization errors were analyzed when new measurement, new sensor, or new anchor is added to the existing sensor networks. In [39], the optimal sensor placement problem was discussed by minimizing the CRLB of the localization error in heterogeneous sensor networks.

However, few of these above-mentioned publications have focused on the error analysis and comparison of the accuracy based on different measurements and algorithms. To address the formerly mentioned crucial problem of algorithm selection and network design based on localization accuracy analysis, in this paper, we contribute to giving the full description of the two typical methods (least-squares and Kalman filter) by using different measurements for target localization in WSNs. We also make theoretical analysis to yield the mean and covariance of the estimation for these different cases while facing three types of measurement noise, which is not related to the true distance between the sensor nodes and the unknown target or related to the distance, respectively. With the theoretical analysis, the optimal sensor placement for the basic 3-sensor case is discussed. Then we compare the accuracy of these different situations through further simulations and analyze the relationship between the accuracy and the detection region. We also simulate the case where the measurement noise is related to the true distance between the multi-sensor nodes and the unknown target. In addition, the errors of 4 typical types of sensor deployment are compared to find out a more reasonable sensor placement in practice.

The error analysis of these two methods (least-squares and Kalman filter) has been made in [41], where we only discuss the case where the measurement noise is not related to the distance and the proof details are not given. This article could be considered an extended version of our conference paper [41]. Compared with [41], the following additional contributions are made in this paper: first, the full mathematical proofs and more discussions are made for least-squares method where the measurement noise is not related to the distance; second, the case where the measurement noise is related to the distance is discussed, especially for Kalman filter method, the estimation and error analysis are sufficiently made for the three models (distance, angle, distance and angle) respectively; in addition, the optimal sensor placement problem is discussed; finally, the comparsion of the simulations and the theoretical analysis have been made to verify the the results obtained.

The remainder of this paper is organized as follows: Problem formulation is first discussed in Section 2 with the description of information of the sensor nodes and their available measurements; Section 3 describes the least-squares and Kalman filter algorithms for localization with different measurements, and we present theoretical analysis for the methods mentioned above based on three types of noise with different characteristics in Section 4; Section 6 gives the optimal sensor placement for basic 3-sensor case under $LS - l$ model; Section 7 shows the simulation results for performance comparison; finally, some concluding remarks are given in Section 8.