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Combining stochastic geometry and statistical mechanics for the analysis and design of mesh networks

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ABSTRACT

We consider a two-dimensional mesh network comprising several source-destination pairs, each communicating wirelessly in a multihop fashion. First, we introduce a novel transmission policy for multihop networks according to which all the buffering in the network is performed at source nodes while relays just have unit-sized buffers. We demonstrate that incorporating this buffering scheme in conjunction with minor amendments to the medium access control (MAC) layer yields several benefits such as keeping packet delays small and helping regulate the traffic flow in a completely distributed fashion. Second, we employ a novel combination of tools from stochastic geometry and statistical mechanics to characterize the throughput and end-to-end delay performances of multihop wireless networks for two different channel access mechanisms, Carrier Sense Multiple Access (CSMA) and ALOHA. Our study also offers valuable insights from a system design stand-point such as determining the optimum density of transmitters or the optimal number of hops along a flow that maximizes the system's throughput performance. We corroborate our theoretical analyses via simulations.

1. Introduction

A mesh network is typically formed by randomly deploying nodes that possess self-organizing capabilities and generally consists of several source-destination pairs communicating wirelessly with each other in a decentralized fashion. Multihop routing, where relays assist in the delivery of packets from the sources to the destinations, is the preferred communication strategy in these networks since it helps conserve battery life and efficiently deliver packets over nodes that are far apart. Mesh networks are extremely desirable for several reasons such as being easily and rapidly deployable and reconfigurable, and also for the fact that they lack single points of failure compared to traditional network architectures with infrastructure. However, inherent technical difficulties have stunted the progress from the era of tetherless connectivity predominated by centralized networks such as cellular telephony and wireless local area networks (WLANs) to the era of ubiquitous wireless connectivity predominated by decentralized mesh networks [1]. We describe the main roadblocks in this regard.

First, while classical information theory has been extremely successful for studying point-to-multipoint links, it is not yet developed enough to characterize the intricacies of multipoint-to-multipoint networks that arise due to the inherent interactions between nodes. In fact, the capacity of a general relay channel with just three nodes is still an open problem. Second, due to the multihop nature of data communication in a mesh network, the flows across various links are spatially and temporally correlated, which needs to be explicitly considered during their analysis and design. Queueing theory has proven useful in this regard, but the analysis gets very cumbersome as the network size grows. Third, in order to optimize the performance of mesh networks, a cross-layer design approach needs to be adopted wherein the interdependencies

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among the layers of the protocol stack, in particular the routing and medium access control (MAC) layers, must be taken into account [2]. Furthermore, the design needs to be adaptive to changes in the system. Owing to such barriers, the performance of general mesh networks has not yet been quantified (beyond scaling laws), and optimal ways of designing and operating them are known only for a few specific and/or simple cases.

In view of these difficulties, researchers are turning to other branches of study to obtain ideas and methodologies that help better understand and characterize the dynamical behavior of wireless networks. Of late, statistical physics has, in particular, captured the attention of the research community since it contains a rich collection of mathematical tools and methodologies for studying interacting many-particle systems. Statistical physics methodologies such as the mean field theory have been employed to study coding over multiuser multiple-input multiple-output (MIMO) channels [3,4]. In [5], the authors have used ideas such as the replica method to characterize the performance of multiuser detection in Code Division Multiple Access (CDMA). The statistical mechanics of interfering transmissions in wireless networks has been proposed in [6,7]. Tools from statistical physics have also been successfully applied to study interesting problems in random communication networks such as percolation, connectivity and capacity [8].

Along similar lines, we employ a novel combination of two new analytical tools, *stochastic geometry* [9] and the *totally asymmetric simple exclusion process* (TASEP) [10], a model in statistical mechanics, to study and design interference-limited wireless networks. Our contributions are twofold:

1. We propose a novel transmission for multihop networks according to which all the buffering is performed at the source nodes while relay nodes have buffer sizes of unity. We demonstrate via simulations that this scheme keeps packet delays small and helps regulate traffic in a completely distributed fashion.
2. We characterize the throughput and end-to-end delay performance of the network for two different channel access schemes using results from stochastic geometry and the TASEP literature. We also provide some insights on optimizing the *throughput density* (to be defined later) in multihop wireless networks that are useful from a design stand-point. Additionally, we validate our analysis with simulations.

To the best of our knowledge, this is the first attempt at combining ideas from stochastic geometry and statistical mechanics.

2. System model

2.1. Network geometry

We consider a mesh network comprising an infinite number of source nodes, each of which initiates a (in general, multihop) flow of packets to a certain (destination) node lasting over an infinite duration of time. This framework is suitable for modeling mesh networks since the aggregate traffic in a mesh network can always be decomposed into several multihop flows. The distribution of source nodes is assumed to be a homogeneous Poisson point process (PPP) on the infinite plane $\mathbb{R}^2$ with density δ. Additionally, the network consists of a countably infinite population of other nodes (potential relays and destinations) arranged as a homogeneous PPP with density $1-\delta$. Thus, the total density of the network is (without loss of generality) equal to unity. For each source node, the destination node is chosen at a random direction, and at a finite distance.

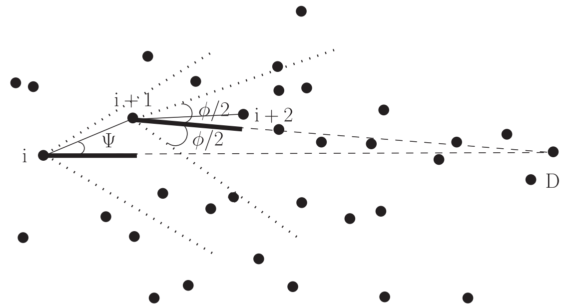

Fig. 1. Illustration of nearest-neighbor routing in a sector of angle $\pm\phi/2$ along the axis to the destination for an arbitrary flow. The packet is routed from node i to node $i+1$, which then relays it to node $i+2$. We denote the argument to the destination by the random variable Ψ. The thick solid lines along the axes to the destination represent the *progress* (to be defined later) of packets across the links $i \to i+1$ and $i+1 \to i+2$.

2.2. Routing strategy

We take that packets are then routed in a general manner as follows.[1] Each node that receives a packet relays it to its n^{th}-nearest neighbor ($n \geqslant 1$) in a sector of angle $\phi \in [0,\pi]$, i.e., the next-hop node is the nth-nearest neighbor that lies within $\pm\phi/2$ of the axis to the destination. Fig. 1 illustrates the case of nearest-neighbor routing ($n=1$).

A sample realization of the system model comprising several source-destination pairs is shown in Fig. 2 with $\delta = 0.05$ and $\phi = \pi/2$. In the figure, each destination node is taken to be located five nearest-neighbor ($n=1$) hops away from its corresponding source, at a random direction.

Note that in this setup, the same common relay node may be a part of multiple flows, in particular when δ is not small.

2.3. Channel model

We consider the case where all nodes use the same frequency band such that simultaneous transmissions cause interference between links. Furthermore, we assume that the transmit power at each transmitting node is equal to unity. Also, we model the attenuation in each link as the product of a large-scale path loss with exponent γ and a

[1] For implementation, each source needs to know its own location and the direction towards its intended destination.