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Graph-based criteria for spectrum-aware clustering in cognitive radio networks

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ABSTRACT

Cognitive radios (CRs) can exploit vacancies in licensed frequency bands to self-organize in opportunistic spectrum networks. Such networks, henceforth referred to as cognitive radio networks (CRNs), operate over a dynamic bandwidth in both time and space. This inherently leads to the partition of the network into clusters depending on the spatial variation of the primary radio network (PRN) activity. In this article, we analytically evaluate the performance of a new class of clustering criteria designed for CRNs, which explicitly take into account the spatial variations of spectrum opportunities. We jointly represent the network topology and spectrum availability using bipartite graphs. This representation reduces the problem of spectrum-aware cluster formation to a biclique construction problem. We investigate several criteria for constructing clusters for the CRN environment, and characterize their performance under different spectrum sensing and PR activity models. In particular, we evaluate the expected cluster size and number of common idle channels within each cluster, as a function of the spectrum and topology variability. We verify our analytical results via extensive simulations.

1. Introduction

Under a fixed spectrum allocation paradigm, frequency bands are licensed for exclusive use and, in many cases, to specific entities. For example, TV bands are used for the broadcast of TV signals from licensed operators, while public safety radio bands are reserved for radio communications of state, governmental, and municipal entities. This paradigm increases the robustness of wireless services by preventing signal interference between different technologies [1,2]. However, measurements of the activity load on the licensed spectrum have shown that a large portion of it is heavily underutilized [3,4]. To this effect, the Federal Communications Committee (FCC) has recently decided to open up part of the spectrum for unlicensed opportunistic access [5].

Policy regulations dictate that opportunistic users must not interfere with the transmissions of legacy systems [5]. This "no interference" policy leads to a hierarchical network architecture in which licensed users, typically referred to as *primary users* or *primary radios* (PRs), have a higher priority in accessing the spectrum compared to unlicensed ones, commonly referred to as *secondary users*. Cognitive radios (CRs) are one of the most promising technologies for implementing the mandated policy regulations [6]. Using software defined radio technology and an advanced cognition engine, CRs are capable of sensing the idle spectrum either independently, or cooperatively [7–9]. The idle spectrum is then temporarily accessed by the CRs to form a cognitive radio network (CRN).

The unique characteristic of a CRN co-existing with a primary radio network (PRN) is the dynamic nature of the spectrum availability [10]. Consider, for example, the co-existence of a PRN with a CRN, as shown in Fig. 1a. PRN traffic variations lead to a spatial and temporal variation of the CRN topology. Two CR nodes within

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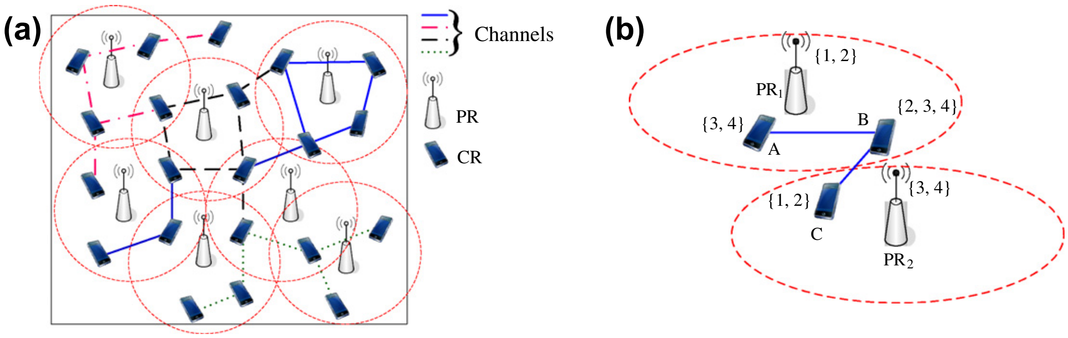

Fig. 1. (a) Co-existence of a CRN with a PRN. The frequency bands exploited by CRs vary in space depending on the ongoing PR activity, (b) the topology of the CRN is dependent on the PR activity. CR nodes *A*, *C* cannot communicate because they do not share a common idle channel, despite being within communication range.

communication range are not guaranteed to communicate, unless at least one idle band exists at their location. This additional constraint imposes an inherent partition of the CRN into clusters, depending not only on the physical proximity of CRs, but also on the spectrum availability. In this article, *we develop and study the performance of clustering criteria that explicitly take into account the spatial variations of the spectrum opportunities.*

We note that co-located CRs make correlated sensing observations by sampling the transmission activity of nearby PRs. The set of idle channels[1] sensed by neighboring CRs varies depending on: (a) the proximity of each CR to active PRs, and (b) the imperfections of the sensing mechanism due to hardware limitations and phenomena of shadowing and fading [10]. For instance, in Fig. 1b, we show three CRs opportunistically accessing a set of four licensed channels. PR_1 occupies channels $\{1,2\}$ while PR_2 occupies channels $\{3,4\}$. CRs *A* and *B* are within the coverage range of PR_1 while CR *C* is within the coverage range of PR_2. CRs *A* and *B* sense no PR activity on channels $\{3,4\}$ while *C* senses no PR activity on channels $\{1,2\}$. In addition, *B* is perceiving channel 2 as idle due to multipath or fading effects. In the CRN of Fig. 1b, *A*, and *C* cannot directly communicate despite the fact that they are within communication range, because there is no overlap between their respective sets of idle channels.

From the example of Fig. 1b, it becomes evident that the network topology jointly depends on the physical proximity and spectrum availability. Therefore, topology management algorithms such as clustering, must take both these parameters into account. However, we make the observation that clustering criteria designed for CRNs with dynamic spectrum, may have conflicting goals. On one hand, partitioning the network to a small number of clusters (with larger cluster sizes) reduces the overhead for topology management [11]. On the other hand, grouping a large number of CRs with dissimilar sets of idle channel, reduces the available bandwidth for intra-cluster communication (a smaller number of idle channels is common among all CRs). To capture the aforementioned trade off, we jointly model the physical network topology and spectrum availability at each CR as a bipartite graph. Based on this joint representation, we partition the CRN into clusters by constructing biclique graphs (complete subgraphs of a bipartite graph), which satisfy various design criteria. We initially proposed the idea representing clusters in CRNs as bicliques in [12]. The goal of the work in [12] was to locally allocate common control channels for coordination purposes. The differences between [12] and the present work are summarized in the following contributions.

Contributions. Adopting a graph-based representation of the idle spectrum, we examine three clustering criteria, suitable for CRNs with dynamic spectrum. These criteria are: (a) joint maximization of the *sum* of common idle channels per cluster with the number of cluster members, (b) joint maximization of the *product* of common idle channels per cluster times the number of cluster members, and (c) maximization of the number of cluster members under a constraint on the number of common idle channels. We show that our clustering criteria can be combined with clustering algorithms proposed for ad hoc networks, in order to perform spectrum-aware distributed clustering in CRNs. Such clustering, not only allows for enhanced intra-cluster communication due to the availability of multiple common frequency bands, but also inherently implements cooperative spectrum sensing. For each clustering criterion, we analytically evaluate the clustering performance in terms of the feasible clusters, the expected cluster size and the number of common idle channels per cluster. In our derivations, we consider two PR activity models; a semi-Markov ON/OFF model and a Poisson traffic model. However, other traffic models can be incorporated to our analytic results. Furthermore, we consider the clustering process under both perfect and imperfect channel state information. Note that our theoretical evaluation entails the estimation of the feasible bicliques that can be constructed from bipartite graphs with a pre-specified probabilistic structure. Our derivations can be applied to any problem that benefits from a mapping to a biclique representation, and is subject to similar probabilistic models.

Paper organization. The remaining of the paper is organized as follows. In Section 2, we state our system model. In Section 3, we develop a graph model for the joint

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[1] In this article, we use the term “channels” to refer to orthogonal frequency bands.