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On convexification of range measurement based sensor and source localization problems

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ABSTRACT

This paper revisits the problem of range measurement based localization of a signal source or a sensor. The major geometric difficulty of the problem comes from the non-convex structure of optimization tasks associated with range measurements, noting that the set of source locations corresponding to a certain distance measurement by a fixed point sensor is non-convex both in two and three dimensions. Differently from various recent approaches to this localization problem, all starting with a non-convex geometric minimization problem and attempting to devise methods to compensate the non-convexity effects, we suggest a geometric strategy to compose a convex minimization problem first, that is equivalent to the initial non-convex problem, at least in noise-free measurement cases. Once the convex equivalent problem is formed, a wide variety of convex minimization algorithms can be applied. The paper also suggests a gradient based localization algorithm utilizing the introduced convex cost function for localization. Furthermore, the effects of measurement noises are briefly discussed. The design, analysis, and discussions are supported by a set of numerical simulations.

1. Introduction

Over the last decade, there has been significant amount of studies on the problem of range or distance measurement based signal source/sensor localization [1–8]. This problem is formulated in abstract terms in [5] as follows:

Problem 1.1. Given known 2 or 3-dimensional sensory station positions $x_1, \ldots, x_N$ ($N > 2$ and $N > 3$ in 2 and 3 dimensions respectively) and a signal source/target at unknown position y^*, estimate the value of y^*, from the measured distances $d_i = \|y^* - x_i\|$.

Problem 1.1 is defined in the form of a cooperative target/source localization task; nevertheless, it can be considered in the form of a sensor network node self-localization problem as well, where the N stations represent N anchors, and there is a $(N+1)$st sensor node at y^* estimating its own position.

The major geometric difficulty of Problem 1.1 comes from the non-convex structure of optimization tasks associated with range measurements: The set of source locations corresponding to a certain distance measurement d_i by a sensor located at point x_i is non-convex both in two and three dimensions, in the form of a circle and a spherical shell, respectively. The generic attempt is then fusing all the distance measurements $d_1, \ldots, d_N$ from the sensing points $x_1, \ldots, x_N$, respectively, and finding the intersection of the non-convex source location sets $S(x_i, d_i)$ corresponding to the (x_i, d_i) pairs. However, the non-convexity of these location sets limits the application of the algorithms devised based on intersection of the mentioned non-convex

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source location sets $S(x_i, d_i)$ and the corresponding non-convex cost functions.

This paper revisits Problem 1.1 following a different approach and suggests a geometric strategy to compose a convex geometric problem first, that is equivalent to the initially non-convex problem, at least in noise-free measurement cases. Once the convex equivalent problem is formed, a wide variety of convex minimization algorithms can be applied. The paper also suggests a gradient based localization algorithm based on the introduced convex cost function for localization. Furthermore, the effects of measurement noises are briefly discussed. The design, analysis, and discussions are supported by a set of numerical simulations.

The details of distance measurement mechanisms used for the above problem is out of scope of this paper. Such details can be found, e.g., in [7,8]. Nevertheless, similar to [5], for better visualization of the implementation of the localization task, we give here one mechanism example, received signal strength (RSS) approach: For a source emitting a signal with source signal strength A in a medium with power loss coefficient η, the RSS at a distance d from the signal source is given by

$$s = A/d^{\eta}. \tag{1.1}$$

Using (1.1), d can be calculated given values of A, s, and η.

The rest of the paper is organized as follows: Section 2 introduces the proposed problem convexification strategy based on the notion of *radical axis*. Section 3.1 proposes a gradient based localization algorithm minimizing the convex cost function introduced in Section 2. Convergence analysis for the noise-free measurement cases is provided in Section 3.2. Simulation studies, including those testing the effects of measurement noises, are presented in Section 4. Closing remarks are given in Section 5.

2. Convexification of the localization problem

2.1. Non-convex cost functions

As stated in Section 1, the approaches to Problem 1.1 in the literature start with a non-convex geometric minimization problem definition and attempt to devise methods to compensate the non-convexity effects. A typical natural selection of cost function to minimize [5] is

$$J_1(y) = \frac{1}{2}\sum_{i=1}^{N}\lambda_i\left(\|x_i - y\|^2 - d_i^2\right)^2, \tag{2.1}$$

where λ_i $(i = 1, \ldots, N)$ are positive weighting terms. A gradient localization algorithm based on minimization of the non-convex cost function (2.1) has been proposed in [5]. Although this algorithm has proven stability and convergence properties, for these guaranteed properties to hold y^* in Problem 1.1 is required to lie in a certain convex bounded region defined by the set $\{x_1, \ldots, x_N\}$. Next we introduce a new cost function to overcome the aforementioned limitation.

2.2. A convex cost function based on radical axes

In two dimensions, if the distance measurements d_i in Problem 1.1 are noise-free, the global minimizer of (2.1) is located at y^*, where $J_1(y^*) = 0$. Geometrically, y^* is the intersection of the circles $C(x_i, d_i)$ with center x_i and radius d_i. We re-formulate this later fact to form a convex cost function to replace the non-convex (2.1), using the notion of *radical axis*:

Theorem 2.1 [9, Fact 45]. *Given two non-concentric circles $C(c_1, r_1), C(c_2, r_2)$, there is a unique line consisting of points p holding equal powers with regard to these circles, i.e., satisfying*

$$\|p - c_1\|^2 - r_1^2 = \|p - c_2\|^2 - r_2^2.$$

This line is perpendicular to the line connecting c_1 and c_2, and if the two circles intersect, passes through the intersection points.

The unique line mentioned in Theorem 2.1 is called the *radical axis* of $C(c_1, r_1)$ and $C(c_2, r_2)$ [9].

Lemma 2.1. *In 2 dimensions, if the distance measurements d_i in Problem 1.1 are noise-free, the intersection set of the radical axes of any $N-1$ distinct circle pairs $C(x_i, d_i), C(x_j, d_j)$ $(i \neq j)$ is $\{y^*\}$.*

Proof. The result straightforwardly follows from Problem 1.1 definition and the last statement of Theorem 2.1. □

In order to utilize Lemma 2.1, we first derive the mathematical representation of the radical axis l_{ij} of a circle pair $C(x_i, d_i), C(x_j, d_j)$ $(i \neq j)$ given the values of x_i, x_j, d_i, d_j. Such a radical axis line is illustrated in Fig. 1. l_{ij} perpendicularly intersects x_ix_j at y_{ij}. Hence any point y on it satisfies

$$(y - y_{ij})^T e_{ij} = 0, \tag{2.2}$$

where

$$e_{ij} = x_j - x_i.$$

It can be observed from Fig. 1 that

$$y_{ij} = x_i + a_i\frac{e_{ij}}{\|e_{ij}\|}, \tag{2.3}$$

as well as $d_i^2 - a_i^2 = d_j^2 - a_j^2 = d_j^2 - (\|e_{ij}\| - a_i)^2$, from which a_i can be calculated as

$$a_i = \frac{\|e_{ij}\|^2 + d_i^2 - d_j^2}{2\|e_{ij}\|}. \tag{2.4}$$

The Eqs. 2.2, 2.3 and 2.4 form the explicit mathematical representation we were looking for.

Next, we focus on utilization of Lemma 2.1 to compose a convex alternative for (2.1). Leaving the optimal selection of the $N-1$ distinct circle (or corresponding sensor node) pairs to a future study, we consider a sequential pair selection for the rest of this paper: For each $i \in \{1, \ldots, N-1\}$, let pair i denote the circle pair $C(x_i, d_i), C(x_{i+1}, d_{i+1})$, l_i denote the corresponding radical axis, y_i denote the intersection of l_i and x_ix_{i+1}; and accordingly let us use the following special case of (2.3) and (2.4):