Journal of Geochemical Exploration 122 (2012) 4–8

Contents lists available at SciVerse ScienceDirect

Journal of Geochemical Exploration

journal homepage: www.elsevier.com/locate/jgeoexp

The fractal relationship between orebody tonnage and thickness

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ARTICLE INFO

Available online 1 July 2012

Keywords:
Tonnage
Thickness
Fractal
Reserve

ABSTRACT

Orebody thickness, grade, cutoff grade and tonnage are the most important parameters in assessing the economic values of ore deposits. Despite abundant research about the relationships among tonnage, grade and cutoff grade, the tonnage–thickness relationship is still undefined in the case of the orebody parameters following fractal distributions. Referring to the deduction from the number-size fractal model to the number-median size model, the median concentration-area (MC-A) model is derived from the concentration-area (C-A) model. Utilizing the C-A model to analyze the plane distribution of orebody thickness (grade·thickness), the orebody area delimited by a given thickness has a fractal relationship with the median thickness (grade·thickness) for that area according to the MC-A model. Ore tonnage, then expressed by the product of the area, median thickness and ore density, has a power-law relationship with the median thickness, as is named tonnage–thickness model. Correspondingly, contained metal tonnage in the ore tonnage shows a fractal relationship with the median grade·thickness, i.e., a metal tonnage–grade·thickness model. The tonnage–thickness model and metal tonnage–grade·thickness model are helpful in understanding orebody spatial distribution, which is demonstrated by a case study from bauxite orebody in Western Guangxi, China.

1. Introduction

Orebody thickness, grade, grade·thickness (product of grade and thickness), cutoff grade and tonnage are the most important parameters used for assessing the economic value of an ore deposit as well as for understanding its genesis. Studies about the parameters can be categorized into different groups. Firstly, some studies involve analysis of (a) cumulative percentage distribution of tonnage or grade of deposits of the same genetic type and clustered in one a region (Singer et al., 2005) and (b) fractal distribution of ore deposit tonnages, orebody thickness and grade in exploration works (Wang et al., 2010a; Zuo et al., 2009a, 2009b). As an example of the latter, Sanderson et al. (1994) have studied the fractal dimension of the distribution of mineralized quartz vein thicknesses and discovered that it changes in relation to gold grade. Secondly, some studies involve analysis of the co-variation (e.g., using a scatter plot) of two selected parameters, mostly tonnage and grade, to show the regional metallogenic features of the studied deposits. Thirdly, some studies involve estimating tonnage–grade–cutoff grade curves through the application of geostatistical simulation methods, fractal simulation methods (Kentwell et al., 1999) as well as explicit equations describing parameter relationships based on fractal models (Wang et al., 2010a, 2010b). Fourthly, some studies involve definition of explicit relationships between tonnages and grades within a set of deposits, like cumulative tonnage–grade relationship (Lasky, 1950) and tonnage–grade fractal relationship (Turcotte, 1997). Finally, some studies involve deriving formulas for the calculation of ore tonnage (or metal tonnage) based on the fractal model distribution of orebody thickness (or grade·thickness) (Wang et al., 2010a, 2011b), yet the formulas are not concise enough to present an inherent relationship for tonnage–thickness or metal tonnage–grade·thickness.

Various fractal models, e.g., number-size (N-S) model and area-concentration (C-A) model, have been proposed and used for quantitative description of the skewed distributions of geological objects (Agterberg et al., 1996; Cheng et al., 1994; Deng et al., 2006, 2008, 2011; Turcotte, 1997; Zuo et al., 2009c, 2009d). The relationship between tonnage and orebody thickness or grade·thickness has not been well clarified with respect to the proposition that orebody parameters have fractal distributions. In this paper, the C-A model (Cheng et al., 1994), which is nowadays often used to analyze geochemical maps (Carranza, 2010a, 2010b, 2011; Deng et al., 2010; Zuo et al., 2009e), is utilized to analyze the plane distribution of orebody thickness (grade·thickness). The median concentration-area (MC-A) model is then derived from the C-A model. The derivation process is similar to that from the N-S model (Mandelbrot, 1983) to the number-average size model proposed by Wang et al. (2011a). Finally, the orebody tonnage–thickness (grade·thickness) model is established according to the MC-A model.

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doi:10.1016/j.gexplo.2012.06.018

2. Methodology

2.1. Basic equations

The N-S model, proposed by Mandelbrot (1983) and widely applied in geology (Cheng, 1999; Deng et al., 2009; Zuo et al., 2009d), is described as:

$$N(\geq r_c) = C_s r_c^{-D_s}; C_s > 0, r_c > 0, D_s > 0, \tag{1}$$

where r_c represents the size of geological objects (e.g., element concentration in ore samples), $N(\geq r_c)$ is the number of objects with sizes equal to or greater than r_c, and C_s and D_s are constant and fractal dimension, respectively.

The number-average size model, which relates the cumulative number $N_i(\geq r_{ci})$ of objects greater than r_c to the average size r_m of such objects (Wang et al., 2011a), is expressed by the following power-law relationship:

$$N(\geq r_c) = C_m r_m^{-D_m}, r_c << r_{max} \tag{2}$$

where C_m is a constant, and D_m is the fractal dimension. The D_m in Eq. (2) is equal to D_s in Eq. (1) when $D_s > 1$; and $D_m = 1$ when $D_s < 1$ (Wang et al., 2011a). Considering that orebody parameters follow certain fractal models, the median is adopted here to replace the average, because the median is a more robust estimate of central tendency when the distribution of values is skewed with a small number of very high or low values, and it is less sensitive to the extreme values than the mean is. Therefore, it is considered here that the number-average size model should be better replaced by the number-median size (N-MS) model.

As the C-A model is applied in a geochemical map, the area $A(r \geq r_c)$ enclosed by contours with elemental concentrations r_c has a power-law relation with the r_c as follows:

$$A(\geq r_c) = C_c r_c^{-D_c} \tag{3}$$

where C_c and D_c are constant and fractal dimension, respectively. As the C-A model is utilized to analyze orebody variables (Wang et al., 2011a), r_c represents orebody thickness t_i or grade·thickness l_i. In this paper, although r_c does not denote the element concentration as in a geochemical map, Eq. (3) is still called a C-A model for consistency with the concept defined by Cheng et al. (1994).

Based also on the C-A model, Wang et al. (2011a) derived a model for ore reserve estimation, named FMRE-CA (fractal model for reserve estimation). With the FMRE-CA model, ore tonnage O can be estimated as:

$$O = \frac{\rho C_c D_c}{1-D_c}\left[t_{max}{}^{1-D_c} - t_{min}{}^{1-D_c}\right](D \neq 1) \tag{4}$$

where ρ is ore density and considered as a constant in this paper, t_{min} and t_{max} are minimum and maximum orebody thickness, respectively. And metal tonnage M can be calculated as:

$$M = \frac{\rho C_c D_c}{1-D_c}\left(l_{max}{}^{1-D_c} - l_{min}{}^{1-D_c}\right)(D \neq 1) \tag{5}$$

where l_{min} and l_{max} are minimum and maximum orebody grade·thickness, respectively.

2.2. Model derivation, utilization and comparison

Derivation of a tonnage–thickness (grade·thickness) model is done (a) in a vertical longitudinal projection (VLP) if an orebody dips more than 45° or (b) in a horizontal longitudinal projection (HLP) if an orebody dips less than 45°. The following mathematical modeling is performed in a VLP. The same process would be applicable in the case of HLP. In a VLP, the horizontal orebody thickness and the grade in each exploratory work (drill hole, test pit, etc.) are calculated according to the cutoff grade and minimum mining thickness, and the corresponding grade·thickness is obtained. The orebody outline is then delimited so as to estimate its area. Based on the discrete exploratory works, contour maps of mineralization variables are obtained via the inverse distance weighting (IDW) interpolation. According to the C-A model of a contour map, the orebody area enclosed by a given contour of thickness (or grade·thickness) shows a fractal relationship with the mineralization parameter.

If the N-S model is applied, a fractal relationship between the cumulative number of objects larger a certain size r_c and the median

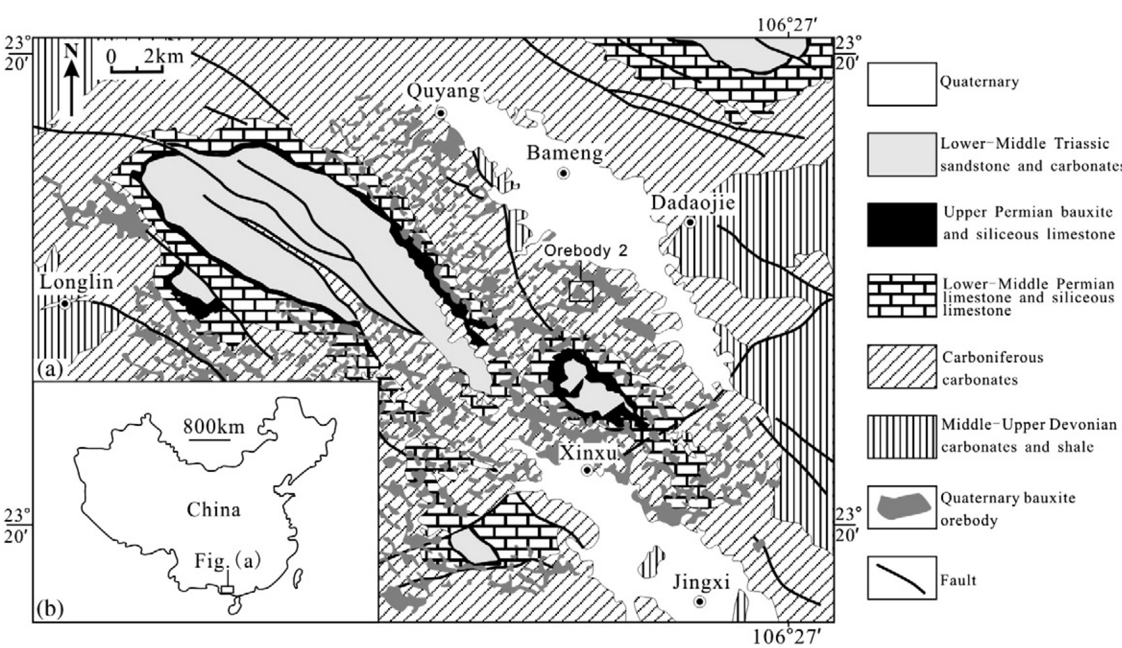

Fig. 1. Simplified geological map of the Xinxu bauxite ore deposit, Western Guangxi, China.