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# Synthesis of electrical oscillators based on principles of symmetry and reflection

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ABSTRACT

The paper discusses the possibility to apply principles of symmetry and reflection for analyzing and synthesizing oscillators of electrical signals. It is established that the principle of symmetry is better to be used for synthesizing oscillating systems based on two-port elements, while the principle of reflection – for synthesizing systems based on one-port elements. The synthesis of oscillating systems using the principle of symmetry is shown to be realized by inverting certain operators of elements of the oscillation system: complex transfer or amplitude characteristics of two-port elements. The synthesis of oscillating systems using the principle of reflection is performed by mirroring amplitude admittance characteristic of a linear one-port element into amplitude admittance characteristic of a nonlinear one-port element. The proposed approach is demonstrated for synthesizing oscillating systems of electrical oscillators; however, it can be applied to analyze any cyclic process.



## 1. Introduction

Principles of symmetry and reflection are widely used in geometry, differential equations theory, physics [1,2], etc. Mutually inverse transformations of differentiation and integration in mathematics, the well-known principle of action and reaction in mechanics, the equality of angles of incidence and reflection in optics can be mentioned in this context. The principle of symmetry enables the analytical construction of linear regulators in the control theory [3]. Some applications of the principle of symmetry for synthesizing structures of measuring converters are considered in [4]. As applied to instrumentation technology, these principles allow one to improve accuracy of signal conversion [5]. They help to explain known and to create new solutions, as well as to discover new regularities in any field of natural science. This paper discusses how the principles of symmetry and reflection can be applied for analyzing and synthesizing electrical oscillators, and substantiates the reproduction of periodic oscillations by such devices.

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## 2. Synthesis of converter structures based on the principle of symmetry

Let us consider the structure shown in Fig. 1a. Values $x_1$ to the left and $x_2$ to the right are any physical quantities, including voltages and currents. These values are connected by direct and inverse equations:

$$\begin{cases} y = Ax_1; \\ x_1 = A^{-1}y; \end{cases} \qquad \begin{cases} x_2 = By; \\ y = B^{-1}x_2. \end{cases} \tag{1}$$

In case of $x_1 = x_2$ and the inverse conversion operator $B$ is equal to $A^{-1}$ ($B = A^{-1}$), the equations of inverse converter (A) are identical in form to the equations of direct converter (B). Moreover, if we send the signal from $A$ output to $B$ input, we get the exact copy of $A$ input signal at the $B$ output. Fig. 1 shows such a case where values $x_1$ to the left are the single-valued mapping to $x_2$ to the right. If $AA^{-1} = E$ or $A^{-1}A = E$, i.e. at one-one operators $A$ and $A^{-1}$, we have $x_1 = x_2$, i.e. each element of set $X_1$ to the left is equal to the corresponding element of set $X_2$ to the right.

It would be mentioned that the blocks A in Fig. 1a may be three-terminal two-ports or four-terminal two-ports with directional conversions. Blocks A in Fig. 1b is may be two-terminals (two

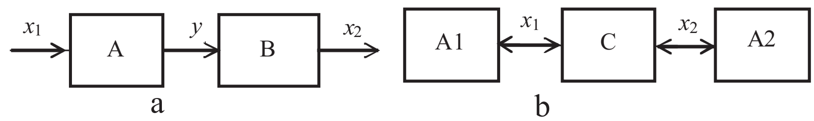


**Fig. 1.** Mutually inverse conversions (a), mutually inverse reflections (b) (C-block of reflections).

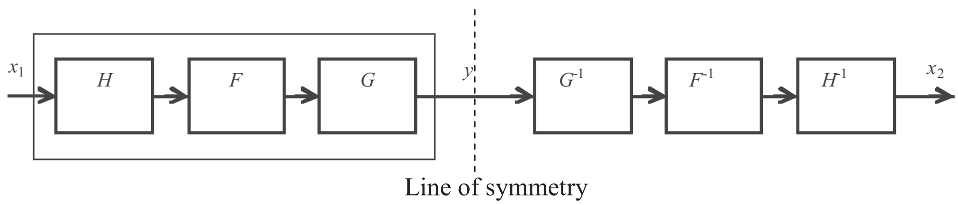


**Fig. 2.** Structures of direct and inverse conversions.

poles) and block C may be bidirectional three-terminal two-ports or four-terminal two-ports.[1]

## 3. Methods of the synthesis based on the principle of symmetry

Let us call the direct synthesis method in which the direct converter structure is found by a known direct equation. The synthesis of the structure can be defined by the following steps:

- The operator $A$ is decomposed into elementary operators. For this purpose, the first equation of the first system (1) is represented as cascade connection of blocks as shown in Fig. 2;

$$y = G(F(\ldots(H(x_1)))); \tag{2}$$

- A nesting sequence of operators is defined;
- Each elementary operator is assigned to the block that performs the relevant operation;
- The blocks are connected with each other according to the nesting sequence of operators.

These steps are shown in the left block of Fig. 2. These actions are seen to implement the method of synthesis based on direct conversion. Let us consider the more formalized method. Suppose that the equation of direct conversion can be represented as (2). Then, we can get the equation of inverse conversion through the formal conversion (see the right block of Fig. 2).

$$x_2 = H^{-1}(\ldots(F^{-1}(G^{-1}(y)))). \tag{3}$$

Note that the input the output values $x$ and $y$ in Eq. (3) are connected by a composition of operators which are inverse to operators (2) and nested in each other in the inverse order.

## 4. Synthesis of oscillator structures based on the principle of symmetry

The block diagram of conversions shown in Fig. 1 is directly related to synthesizing circuits of oscillating systems (OS) for signal oscillators since, when the output $x_2$ and input $x_1$ are connected, the diagram is closed in a ring and, at certain mutually inverse operators, becomes an oscillating system. Note that operators can be represented by complex frequency or amplitude characteristics of the blocks, etc. Fig. 3a shows the classical structure of the oscillator based on active nonlinear element (ANE1) of voltage-controlled voltage source (VCVS) type and the linear frequency-dependent circuit (LFDC).

Let us assume that the waveforms of input $x_1(t)$ and output $y(t)$ signals are known. In this case, the conversion characteristic (LFDC) can be described by the equation $\gamma(x_1) = y$. This characteristic shows how instantaneous values of the output circuit signal depend on the instantaneous values of the input signal. Let us call it the amplitude characteristic. Similarly, amplitude characteristic of ANE can be represented by another equation $K(y) = x_2$. Then, conversion of the second equation gives $K^{-1}(x_2) = y$. Taking into account that in stationary mode, when periodic oscillations are generated, $x_1 = x_2 = x$, the characteristics of LFDC and ANE are related to each other as follows:

$$K^{-1}(x) = \gamma(x). \tag{4}$$

As follows from (4), the amplitude characteristics of LFDC and ANE must be mutually inverse for generating oscillations of a specified periodic waveform. In Eq. (4), signals $x(t)$ and $y(t)$ are represented by voltages. Therefore, ANE can be implemented on the basis of voltage controlled voltage sources (VCVS). However, similar equations can be written for other controlled sources, e.g. $K_i^{-1}(x) = \gamma_i(x)$ – for current-controlled current source (CCCS), $Z^{-1}(x) = G(x)$ – for current-controlled voltage source (CCVS) and $G^{-1}(y) = Z(y)$ – for voltage-controlled current source (VCCS).

Complex frequency characteristics can be considered in the similar way. For example, if operator of $\dot{\gamma}(\omega) = \gamma(\omega)e^{j\varphi_\gamma}$ is a complex frequency characteristic of a RC or LC circuit (LFDC), the inverse operator of $\gamma^{-1}(\omega) = K(\omega)e^{j\varphi_k}$ (also the complex frequency characteristic) of the ANE must correspond to the active element. It is

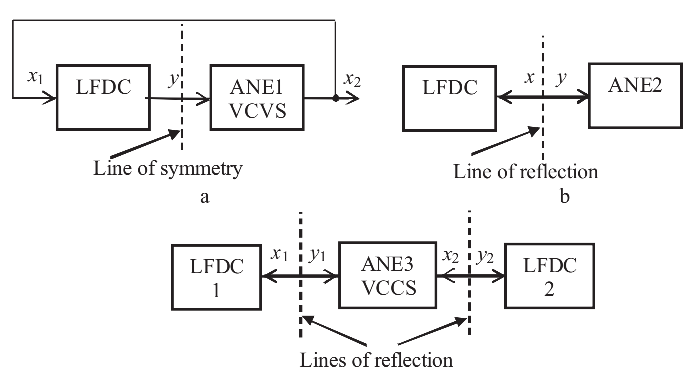


**Fig. 3.** Structural models of OS based on active nonlinear elements with unidirectional (a) and bidirectional (b and c) signal transmission.

[1] Note that when you close the loop the loop-gain is one and the phase-shift is zero also in the case where the open loop does not have gain one and phase-shift zero.