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A modified variational functional for estimating dense and discontinuity preserving optical flow in various spectrum

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ABSTRACT

Variational methods are among the most successful approaches in the estimation of optical flow between two images. This paper presents a variational energy functional that incorporates the global model of Horn and Schunck (1981) and the classical model of Nagel and Enkelmann (1986) as a new regularization functional. In particular, the objective of this paper is to combine the advantages of both these models. This formulation yields a dense optical flow, preserves discontinuities in the flow field, and provides a significant robustness against outliers (occlusions, illumination changes and noise). The proposed variational functional is solved by an efficient numerical scheme. Stability and convergence analysis are given in order to show the mathematical applicability of the proposed model. Experimental results on different datasets verify the robustness and accuracy of the proposed model.

1. Introduction

Recovery of optical flow from a sequence of images is a challenging problem in computer vision/image processing. Optical flow is defined as a two dimensional (2D) velocity vector, which arises either due to the motion of the objects in the scene or by the motion of the observer/camera. The objective of its estimation is to determine a displacement between two images. Optical flow furnishes the dynamic information of an object between the frames of a scene, i.e., how many units the pixel/object has been moved compared to the previous frame. It has several applications in vision system such as 3D reconstruction, automatic navigation, video surveillance, human action understating, medical diagnosis [1–3]. Extraction of the optical flow has been also used as a preprocessing step in many vision algorithms such as visual control, motion parameter estimation, image segmentation, etc.

Estimation of the optical flow is considered as an ill-posed problem. Thus, some additional constraints are required in order to regularize the flow field during optical flow estimation. Many different models have been proposed to estimate the optical flow starting from the seminal work [4,5] and attained an impressive level of accuracy and performance. A global approach proposed by Horn and Schunck [4] yields a dense flow, but it is experimentally sensitive to noise. A local approach proposed by Lucas and Kanade [5] is robust under noise, but it is unable to yield dense flow.

Differential variational models are considered as successful approaches for the estimation of optical flow. These approaches achieved a good attention of researchers in the recent past [6–9]. The reasons are simplicity in modeling and good quality results. The quality of an optical flow estimation model is judged based on accuracy, discontinuity in the flow field and large motion. In order to produce a stable motion, a larger region of integration is more desirable. However, it may possible to have multiple motions of the objects. Therefore, the estimation of a dense and discontinuity preserving optical flow is a challenging task. To achieve this, many assumptions are considered in different variational models to get better performances. Recent models like [10,11,9] include the additional constraints such as gradient constancy assumption and the convex robust data term to obtain the impressive level of performance under outliers, noise and to avoid the problem of local minima. Many of the variational models such as [6,7,12,13,8,14–16] considered different constraints or regularizers to get a piecewise optical flow and preserves discontinuity at the boundaries. Moreover, motion segmentation and parametric models have been proposed to break the motion field into several smooth and piecewise parts [17,18,6,13]. Nevertheless, these models still have lacked to provide dense and discontinuity preserving optical flow simultaneously due to the inherent characteristics of the parametric models. More recently, Weinzaepfel et al. [19] proposed a descriptor matching algorithm, named as deep matching for optical flow estimation

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in the variational approach, which boosts the performance of the model for large motions. The deep matching algorithm is based on a multi-stage architecture with six layers interleaving max-pooling and convolutions, a construction akin to deep convolutional nets. In [20], Martinel et al. were given a brief overview of the brain-inspired learning architectures consisting of artificial neural networks (ANNs), neural trees (NTs), convolutional neural networks (CNNs) and extreme learning machines (ELMs) as an introductory part. They proposed a new architecture that borrows the strengths of ANNs, CNNs, NTs and ELMs, and combines them into a unique system, which is named as extreme deep learning tree (EDLT).

In this study, we provide a more efficient variational model to estimate the optical flow. We consider a variational energy functional which combines the global model of Horn and Schunck [4] and the classical model of Nagel and Enkelmann [8]. This helps to lead a dense flow over a region and preserves discontinuities in the optical flow. The formulated variational functional results into a more efficient numerical scheme. The performance of the proposed model is evaluated on various spectrum image sequences, and compared with some existing models. The convergence analysis of the numerical scheme is provided to support the applicability of the model. The robustness of the proposed model are tested in the presence of noise. The performance of the model is also tested under different parameter settings.

The rest of the paper is organized as follows: Section 2 describes the optical flow constraints and the modified variational energy functional with related Euler–Lagrange equations. Section 3 describes the numerical solution and the convergence analysis of the scheme. Section 4 describes the experimental datasets and evaluation methods followed by the experimental results. Conclusions are given in Section 5.

2. Mathematical formulation of variational optical flow model

2.1. Optical flow as variational problem

One of the key assumptions introduced in the estimation of optical flow is the constancy of image gray levels [4]. Let $I(x, y, t)$ and $I(x+\delta x,\ y+\delta y,\ t+\delta t)$ be the gray levels of a pixel in two consecutive frames taken at a time interval δt and positions (x, y) and $(x+\delta x,\ y+\delta y)$, respectively. Then by the constancy of gray levels assumption, we have

$$I(x+\delta x, y+\delta y, t+\delta t) = I(x, y, t) \tag{1}$$

where $I : \Omega \subset \mathbb{R}^3 \to \mathbb{R}$ represents a rectangular image sequence.

Using Taylor series expansion of the left-hand side of (1) and neglecting the second or higher order terms, we have

$$uI_x + vI_y + I_t = 0 \tag{2}$$

where $u = \frac{\delta x}{\delta t}$ and $v = \frac{\delta y}{\delta t}$ are the components of velocity in x- and y-directions, respectively. The terms I_x, I_y and I_t denote the partial derivatives of the intensity w.r.t. x, y and t, respectively. The notations $u(x, y)$ and $v(x, y)$ are called the optical flow in x- and y-directions, respectively. The expression derived in (2) is known as the optical flow constraint (OFC), and demonstrates an important role in the optical flow estimation. The solution of the problem optical flow estimation is under-determined and therefore an extra constraint on (u, v) is required in order to have a unique solution. This constraint is known as smoothness constraint and tells that neighboring pixels should have the uniform flow field in a small area.

The seminal work of Horn and Schunck [4] minimizes the following energy functional to estimate the optical flow:

$$E(w) = \int_\Omega \left[(uI_x + vI_y + I_t)^2 + \alpha^2 \left\{ \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 \right\} \right] dxdy \tag{3}$$

where $w := (u, v, 1)^T$. The first and second terms in the above energy functional (3) are called the data and smoothness terms, respectively. This smoothness term is controlled by a regularization parameter $\alpha(>0)$. The larger the value of α in (3), leads to the smooth optical flow field. Therefore, if $\alpha \gg 0$, then (3) reduces to the following functional:

$$E(w) = \int_\Omega \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 \right] dxdy \tag{4}$$

It means that more influence is given to the smoothness constraint. In a similar way, the smaller the value of α in (3), leads to a continuous optical flow field. Therefore, when $\alpha \to 0$, then (3) reduces to the following functional:

$$E(w) = \int_\Omega (uI_x + vI_y + I_t)^2 dxdy \tag{5}$$

This shows that the total influence is given to the optical flow constraint.

2.2. Proposed variational optical flow model

In order to find much more intuitive understanding and improve the robustness of the variational model, a modified version of the energy functional (3) is defined as

$$E(w) = \int_\Omega \left[(uI_x + vI_y + I_t)^2 + \epsilon^2(u^2 + v^2) + \alpha^2 \left\{ \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 \right\} \right] dxdy \tag{6}$$

where ϵ is a small positive constant. Our main objective is to minimize the above variational energy functional (6). The motivation behind the variational energy functional (6) containing the additional square term is that to keep the minimization scheme simpler and increase the robustness of the method.

The proposed energy functional (6) is the combination of the models [4,8]. The model of [4] provides dense and smooth flow inside each motion field, but unable to preserves discontinuity in the optical flow, and therefore flow propagates in all the directions. The model [8] provides more sharp edges and boundaries in the flow fields, and also preserves discontinuity in the optical flow. Thus, it fuses the advantages of each of them. The advantages to consider this additional term in (6) with respect to (3) can be summarized into the following aspects:

- Given energy functional combines the advantages from [4] as well as [8].
- Provides dense and smooth flow over each motion field, and avoids the propagation of the optical flow from one region to another.
- Efficiently preserves discontinuity in the optical flow.