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Lattice structure for arbitrary-length oversampled linear phase paraunitary filter bank with unequal numbers of symmetric and antisymmetric filters☆

Bodong Li [a,b], Xieping Gao [a,b,*]

[a] MOE Key Laboratory of Intelligent Computing & Information Processing, Xiangtan University, Xiangtan 411105, China
[b] College of Information Engineering, Xiangtan University, Xiangtan 411105, China

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ABSTRACT

Oversampled linear phase paraunitary filter bank (OLPPUFB) can be efficiently designed via lattice structure. Xu et al. have studied the lattice structure for arbitrary-length OLPPUFB (ALOLPPUFB), i.e. OLPPUFB with filter length $KM+\beta$, where M is the integer decimation factor, K is an integer, and β is an integer between 0 and M. Such work was restricted to be used for the case with equal numbers of symmetric and antisymmetric filters, and cannot be easily generalized for other possible cases. To address this issue, we develop in this letter the lattice structure for ALOLPPUFB with unequal numbers of symmetric and antisymmetric filters. The proposed method is carried out by combining the polyphase matrices of OLPPUFB with filter length KN, where N is the integer decimation factor, K is an integer. The efficiency of the method is shown by design examples.

1. Introduction

Oversampled filter bank (OFB) has been extensively used in signal processing [1,2]. One of the efficient methods for the design is to employ lattice structure [3–26], which supplies fast and robust implementation, as well as structural possession of important properties such as linear phase (LP) and paraunitary (PU).

Several works have been published for lattice structures of oversampled linear phase paraunitary filter bank (OLPPUFB). For the convenience of later discussion, the OLPPUFB with filter length KM is named constrained-length OLPPUFB (CLOLPPUFB), whereas the one with filter length $KM+\beta$ (i.e. the one discussed in this letter) is called arbitrary-length OLPPUFB (ALOLPPUFB). Up to now, the lattice structure for OLPPUFB is mainly devoted to constrained-length case. In 2000, Labeau et al. [27] established a necessary condition for the existence of CLOLPPUFB on symmetry polarity (i.e. the number of symmetric filters n_s and antisymmetric filters n_a), and designed the lattice structure for the case with $n_s=n_a$ and the case with $n_s=n_a+1$. Three years later, Gan et al. [28] systematically studied the theory and design of CLOLPPUFB. They obtained a more exact necessary condition for the existence of CLOLPPUFB, and developed the lattice structures covering all possible cases of the condition. In contrast to CLOLPPUFB, ALOLPPUFB provides more choices of filter bank and thus offers better trade-off between filter length and filter performance. Nevertheless, only lattice structure for the case with $n_s=n_a$ has been developed [29], and the other cases still leave to be done and cannot be easily generalized from this case.

To handle this problem and provide more choices of ALOLPPUFB, we describe in this letter the lattice structure of ALOLPPUFB for the case with $n_s \neq n_a$. The design is carried out by combining polyphase matrices of CLOLPPUFB, and the result can be used for all possible cases when an ALOLPPUFB exists, no more than only the considered case with $n_s \neq n_a$. Also by combing polyphase matrices of constrained-length filter bank, we [30] have designed the lattice structure for arbitrary-length *critically-sampled* linear phase paraunitary filter bank. Nevertheless, the difference is remarkable. In contrast to [30], the sizes of several blocks in the design in this letter have to be carefully set, and we determine the sizes based on a new proposed lemma.

Some notations used in this letter are described as follows. The ceil and floor of a real number x are represented by $\lceil x \rceil$ and $\lfloor x \rfloor$ respectively. The numbers of symmetric and antisymmetric filters are denoted by n_s and n_a respectively. Define $f(K, s)=\lceil K/2 \rceil(1+s)/2+\lfloor K/2 \rfloor(1-s)/2$, where K is a non-negative integer and $s=\pm 1$; it can be easily checked that $f(K, 1)+f(K, -1)=K$ and $f(K, 1) \geq f(K, -1)$. The symbols $\mathbf{I}$ and $\mathbf{J}$ denote identity matrix and exchange matrix respectively, and subscripts will be given if their sizes are not clear from the context.

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* Corresponding author.
E-mail addresses: bodong.li@gmail.com (B. Li), xpgao@xtu.edu.cn (X. Gao).

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2. Preliminaries

A P-channel filter bank with filter length $KM+\beta$ is an ALOLPPUFB if the associated polyphase matrix satisfies the PU property $\mathbf{E}^T(z^{-1})\mathbf{E}(z)=\mathbf{I}_M$ and the LP property $\mathbf{E}(z)=z^{-(K-1)}\mathbf{D}\mathbf{E}(z^{-1})\hat{\mathbf{J}}(z)$, where $\mathbf{E}(z)$ is the $P\times M$ polyphase matrix, $\mathbf{D}=\mathrm{diag}(\mathbf{I}_{n_s}, -\mathbf{I}_{n_a})$ and is named *symmetry polarity matrix*, $\hat{\mathbf{J}}(z)=\mathrm{diag}(z^{-1}\mathbf{J}_\beta, \mathbf{J}_{M-\beta})$, and $K-1$ is called the order of the system. The ALOLPPUFB is degraded into a CLOLPPUFB if $\beta=0$.

As with CLOLPPUFB [28], the polyphase matrix of an order-$(K-1)$ ALOLPPUFB can be represented as

$$\mathbf{E}(z)=\begin{cases}\mathbf{G}_{K-1}(z)\cdots\mathbf{G}_2(z)\mathbf{G}_1(z)\mathbf{E}_0(z), & n_s=n_a\\ \mathbf{G}_{\left\lfloor\frac{K-1}{2}\right\rfloor}(z)\cdots\mathbf{G}_2(z)\mathbf{G}_1(z)\mathbf{E}_0(z), & n_s\neq n_a\end{cases}\quad(1)$$

where $\mathbf{G}_i(z)$ and $\mathbf{E}_0(z)$ are called *propagating block* and *starting block* respectively. The propagating block satisfies $\mathbf{G}_i(z)=z^{-N_1}\mathbf{D}\mathbf{G}_i(z^{-1})\mathbf{D}$, where $N_1=1$ when $n_s=n_a$, and $N_1=2$ when $n_s\neq n_a$ [28]. The starting block meets $\mathbf{E}_0(z)=z^{-N_0}\mathbf{D}\mathbf{E}(z^{-1})\hat{\mathbf{J}}(z)$, where N_0 is the minimum order of the starting block. One can easily check that, $N_0=0$ when $n_s=n_a$, and $N_0=\mathrm{mod}(K-1, 2)$ when $n_s\neq n_a$. As shown above, the design of $\mathbf{E}(z)$ can be obtained by developing the building blocks $\mathbf{G}_i(z)$ and $\mathbf{E}_0(z)$. The construction of $\mathbf{G}_i(z)$ has been well formulated in Eqs. (8) and (23) in [28]. Hence the key to the design of an ALOLPPUFB is to create the starting block $\mathbf{E}_0(z)$.

Xu et al. [29] have studied the theory and design of ALOLPPUFB. They proposed in [29, Theorem 1] the necessary condition for the existence of ALOLPPUFB on symmetry polarity. The condition was devoted to all possible cases with different (M, β, K)'s, and we reformulate it in Lemma 1 so that one can handle different (M, β, K)'s uniformly.

Lemma 1. *Let $r_0=f(\beta, 1)+f(M-\beta, 1)$ and $r_1=f(\beta, (-1)^K)+f(M-\beta, (-1)^{K-1})$. As to an order-$(K-1)$ ALOLPPUFB, the numbers of symmetric and antisymmetric filters n_s and n_a satisfy $r_0\le n_s\le P-M+r_1$ and $M-r_1\le n_a\le P-r_0$.*

As can be seen from Lemma 1, the upper bounds of n_s and n_a (denoted by $n_{s,u}$ and $n_{a,u}$) can be represented by their lower bounds (represented by $n_{s,l}$ and $n_{a,l}$), i.e. $n_{s,u}=P-n_{a,l}$ and $n_{a,u}=P-n_{s,l}$. We therefore can proceed with the following discussion using only the lower bounds, i.e.

$$n_s\ge r_0,\quad n_a\ge M-r_1.$$

Xu et al. have developed in [29] the lattice structure of ALOLPPUFB for the case with $n_s=n_a$, and the case with $n_s\neq n_a$ still leave to be done and will be discussed in next section.

3. Lattice structure for ALOLPPUFB

Theorem 1. *Suppose that $\mathbf{E}_0^{(0)}(z)$ is the polyphase matrix of an order-(N_0+1) CLOLPPUFB with decimation factor β and symmetry polarity matrix $\mathrm{diag}(\mathbf{I}_{x_0}, -\mathbf{I}_{y_0})$, and $\mathbf{E}_0^{(1)}(z)$ is the polyphase matrix of an order-N_0 CLOLPPUFB with decimation factor $M-\beta$ and symmetry polarity matrix $\mathrm{diag}(\mathbf{I}_{x_1}, -\mathbf{I}_{y_1})$, where $N_0=0$ when $n_s=n_a$, and $N_0=\mathrm{mod}(K-1, 2)$ when $n_s\neq n_a$. Let*

$$\mathbf{E}_0(z)=\mathrm{diag}(\mathbf{Q}_0, \mathbf{Q}_1)\mathbf{P}\mathrm{diag}(\mathbf{E}_0^{(0)}(z), \mathbf{E}_0^{(1)}(z))\quad(2)$$

where $\mathbf{Q}_0$ and $\mathbf{Q}_1$ are $n_s\times(x_0+x_1)$ and $n_a\times(y_0+y_1)$ free paraunitary matrices respectively, and $\mathbf{P}=\begin{bmatrix}\mathbf{I}_{x_0} & & & \\ & & \mathbf{I}_{x_1} & \\ & \mathbf{I}_{y_0} & & \\ & & & \mathbf{I}_{y_1}\end{bmatrix}$. Let $\mathbf{E}(z)$ be represented as in (1), then $\mathbf{E}(z)$ leads to an order-$(K-1)$ ALOLPPUFB.

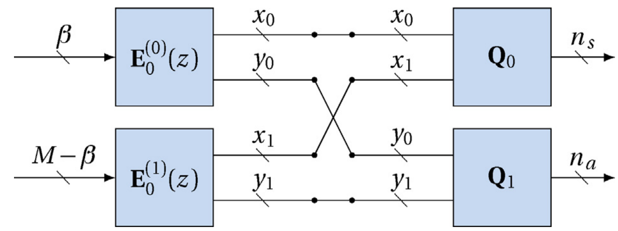

Fig. 1. Lattice structure for the starting block $\mathbf{E}_0(z)$.

Proof. From the fact that each matrix in (2) is PU, $\mathbf{E}_0(z)$ is PU. From the hypothesis about $\mathbf{E}_0^{(0)}(z)$ and $\mathbf{E}_0^{(1)}(z)$, one can obtain

$$\mathbf{E}_0^{(0)}(z)=z^{-(N_0+1)}\mathrm{diag}(\mathbf{I}_{x_0}, -\mathbf{I}_{y_0})\mathbf{E}_0^{(0)}(z^{-1})\mathbf{J}_\beta\quad(3)$$

$$\mathbf{E}_0^{(1)}(z)=z^{-N_0}\mathrm{diag}(\mathbf{I}_{x_1}, -\mathbf{I}_{y_1})\mathbf{E}_0^{(1)}(z^{-1})\mathbf{J}_{M-\beta}.\quad(4)$$

Represent $\mathbf{Q}_0$ and $\mathbf{Q}_1$ into submatrices-form as $\mathbf{Q}_0=[\mathbf{Q}_{00}, \mathbf{Q}_{01}]$, $\mathbf{Q}_1=[\mathbf{Q}_{10}, \mathbf{Q}_{11}]$, where the sizes of the submatrices are $n_s\times x_0$, $n_s\times x_1$, $n_a\times y_0$, and $n_a\times y_1$ respectively. One can easily check that

$$\mathrm{diag}(\mathbf{Q}_{00}, \mathbf{Q}_{10})=\mathbf{D}\mathrm{diag}(\mathbf{Q}_{00}, \mathbf{Q}_{10})\mathrm{diag}(\mathbf{I}_{x_0}, -\mathbf{I}_{y_0})\quad(5)$$

$$\mathrm{diag}(\mathbf{Q}_{01}, \mathbf{Q}_{11})=\mathbf{D}\mathrm{diag}(\mathbf{Q}_{01}, \mathbf{Q}_{11})\mathrm{diag}(\mathbf{I}_{x_1}, -\mathbf{I}_{y_1}).\quad(6)$$

Putting the submatrices-form of $\mathbf{Q}_0$ and $\mathbf{Q}_1$ into (2) leads to

$$\mathbf{E}_0(z)=[\mathrm{diag}(\mathbf{Q}_{00}, \mathbf{Q}_{10})\mathbf{E}_0^{(0)}(z), \mathrm{diag}(\mathbf{Q}_{01}, \mathbf{Q}_{11})\mathbf{E}_0^{(1)}(z)]$$

which, along with (3)–(6), produces $\mathbf{E}_0(z)=z^{-N_0}\mathbf{D}\mathbf{E}_0(z^{-1})\hat{\mathbf{J}}(z)$. From this equation, coupled with the PU property of $\mathbf{E}_0(z)$, one can get that $\mathbf{E}_0(z)$ will yield an order-N_0 ALOLPPUFB.

Since each propagating block $\mathbf{G}_i(z)$ is PU and, as can be easily seen from (1), the sum of the orders of all propagating block is $K-1$ for even M and $2\lfloor K/2\rfloor$ for odd M. Along with value of N_0, one can conclude that $\mathbf{E}(z)$ will produce an order-$(K-1)$ ALOLPPUFB. □

Let us check when we can find an ALOLPPUFB based on Theorem 1. This relies on the existences of the blocks $\mathbf{G}_i(z)$, $\mathbf{Q}_0$, $\mathbf{Q}_1$, $\mathbf{P}$, $\mathbf{E}_0^{(0)}(z)$ and $\mathbf{E}_0^{(1)}(z)$. The blocks $\mathbf{G}_i(z)$ and $\mathbf{P}$ can be found without constraints. The blocks $\mathbf{Q}_0$ and $\mathbf{Q}_1$ exist if

$$n_s\ge x_0+x_1,\quad n_a\ge y_0+y_1.\quad(7)$$

According to [28], $\mathbf{E}_0^{(0)}(z)$ and $\mathbf{E}_0^{(1)}(z)$ can be obtained if

$$x_0\ge f(\beta, 1),\quad y_0\ge\beta-f(\beta, (-1)^{N_0+1})\quad(8)$$

$$x_1\ge f(M-\beta, 1),\quad y_1\ge M-\beta-f(M-\beta, (-1)^{N_0})\quad(9)$$

and their constructions have also been published in [28]. Based on Lemma 2 as shown later, if n_s and n_a satisfy Lemma 1, one can always find x_0, x_1, y_0 and y_1 meeting (7)-(9). Hence Lemma 1 is also sufficient for the existence of ALOLPPUFB. Though we focus on the case with $n_s\neq n_a$ in this letter, the proposed design in Theorem 1 can be used for all possible cases, i.e. the cases restricted by Lemma 1. As mentioned before, the key to the design is to construct $\mathbf{E}_0(z)$, and we depict in Fig. 1 the construction of $\mathbf{E}_0(z)$ obtained using Theorem 1.

Lemma 2. *If n_s and n_a satisfy Lemma 1, then there exist x_0, x_1, y_0 and y_1 meeting (7)–(9).*

Proof. By Lemma 1, we have

$$n_a\ge(\beta-f(\beta, (-1)^K))+(M-\beta-f(M-\beta, (-1)^{K-1}))\quad(10)$$

$$n_s\ge f(\beta, 1)+f(M-\beta, 1).\quad(11)$$

When $n_s\neq n_a$, from the fact that $N_0=\mathrm{mod}(K-1, 2)$, Eq. (10) becomes

$$n_a\ge(\beta-f(\beta, (-1)^{N_0+1}))+(M-\beta-f(M-\beta, (-1)^{N_0})).\quad(12)$$