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# Analysis of envelope correlation on performance of MRC in correlated Rician-fading channels

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ABSTRACT

A simple envelope correlation expression of two maximal-ratio combined (MRC) signals in correlated diversity Rician-fading channels is derived in high signal-to-noise ratios (SNRs). In addition, the relationship between the system performance and envelope correlation is also obtained, which indicates that it is much easier to evaluate the system performance using the envelope correlation, since the effects of parameters such as the Rician factor and all channel attenuations are able to be replaced by the envelope correlation. The validity of the analytical relationship is verified by numerical simulations using a correlated Rician-fading emulator.



## 1. Introduction

Maximal ratio combining (MRC) diversity has been shown to be an effective way to combat multipath fading and interference in various radio communication systems such as MIMO systems, spectrum diversity systems, and satellite communications [1,6,7,9,10]. Unfortunately, channel correlation among the diversity branches would yield a non-negligible degradation [2]. Therefore, researches on the correlation property of MRC outputs and performance of the diversity scheme over correlated fading channels are growing rapidly in radio communication systems.

The channel envelope correlation in the narrow-band system is studied as early as in [3], and then developed over correlated Rician-fading channels in [4]. The power correlation is addressed in [5], and, neglecting the noise, the envelope auto-correlation is analyzed in [6] for the MRC outputs in correlated Rician-fading channels. Recently, [7] investigate the statistical properties of correlated Rician-fading channels, and also analyze the performance of MRC outputs.

On the other hand, relevant researches on the evaluation of system performance in correlated fading channels include [8–18]. Closed-form expressions for the ergodic capacity and outage probability of MRC diversity are given in the case of correlated Rayleigh-fading channels in [8,9] and correlated Rician-fading channels in [10–14]. While [15–18] propose the approximate expressions of system performances in some other correlated fading channels. However, the analytical expressions in above works are always too complex to apply in the practical system, and there is still no relationship found between the envelope correlation and system performance.

In this paper, we address the cross-correlation of MRC outputs considering the noise in correlated Rician-fading channels, and a simple envelope correlation formula is obtained. In addition, the relationship between the envelope correlation and the system performance is derived, which shows that the evaluations of the outage probability and ergodic capacity can be simplified since the envelope correlation is able to replace the effect of parameters such as channel attenuations of all branches and Rician factor, since, in general, the envelop correlation can be directly obtained by the received signals, while the Rician factor and channel attenuations all need the redundant signals to be estimated.

The remainder of this paper is organized as follows. The expression for the envelope correlation is derived and its properties are presented in Section 2. We then investigate the relationship between the envelope correlation and system performance in Section 3. Numerical results and conclusions are given in Sections 4 and 5, respectively.

## 2. The envelope correlation

In this section, we first derive the envelope correlation between two MRC outputs, and then analyze the properties of the envelope correlation in several special cases.

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### 2.1. Envelope correlation derivation

The MRC diversity receiver is employed over correlated Rician-fading channels, and $J$ resolvable branches are assumed to be at each carrier. Then, the received signal on the $k$th branch at carrier $i$ is given by

$$r_{i,k} = \sqrt{P_s} S\alpha_{i,k} + \sqrt{P_n} n_{i,k}, \tag{1}$$

where $n_{i,k}$ is the Gaussian noise with unit variance, $S$ is the normalized transmit signal that $|S| = 1$, $P_s$ and $P_n$ are the signal and noise powers, respectively, $\alpha_{i,k} = (x_{i,k} + m_{i,k}) + j(\bar{x}_{i,k} + \bar{m}_{i,k})$ denotes the Rician-fading channel with scatter part $x_{i,k} + j\bar{x}_{i,k}$ and direct path $m_{i,k} + j\bar{m}_{i,k}$. In general, for a given carrier, the total power of fading branches are assumed to be normalized so that $\sum_{k=1}^{J}\mathbb{E}[|\alpha_{i,k}|^2] = 1$, where $\mathbb{E}[\,\cdot\,]$ is the expectation function.

In general, the corresponding branches of different carriers are set to be correlated. That is [3],

$$\begin{aligned} &Cov[x_{1,k}, x_{2,k}] = Cov[\bar{x}_{1,k}, \bar{x}_{2,k}] = \beta_k,\\ &Cov[x_{1,k}, \bar{x}_{2,k}] = -Cov[\bar{x}_{1,k}, x_{2,k}] = -2\pi\Delta f\tau_{rms}\beta_k,\\ &\beta_k = \frac{\sigma_{1,k}\sigma_{2,k}J_0(2\pi f_m\Delta\tau)}{1 + 4\pi^2\Delta f^2\tau_{rms}^2}, \quad k = 1, 2, \ldots, J, \end{aligned} \tag{2}$$

where $Cov[\cdot]$ denotes the covariance function, $\sigma_{i,k}^2$ is the variance value of $x_{i,k}$ (or $\bar{x}_{i,k}$), $\Delta f$ and $f_m$ are the frequency separation and maximal Doppler shift, respectively, $J_\lambda(\cdot)$ is the $\lambda$ order Bessel function of the first kind, and $\tau_{rms}$ and $\Delta\tau$ are the root mean square delay spread and time delay between two corresponding branches, respectively.

The perfect channel estimation is assumed to be available at the receiver. Then, the MRC output at $i$th carrier can be written as

$$r_i = \sum_{k=1}^{J} r_{i,k}\alpha_{i,k}^* = \sum_{k=1}^{J}\sqrt{P_s}S|\alpha_{i,k}|^2 + \sqrt{P_n}n_{i,k}\alpha_{i,k}^*, \tag{3}$$

where $(\,\cdot\,)^*$ denotes the conjugate.

Considering the high SNR, the envelope of the desired signal is always much larger than that of the noise, that is

$$Pr\left\{\left|\sum_{k=1}^{J}\sqrt{P_s}S|\alpha_{i,k}|^2\right| \gg \left|\sum_{k=1}^{J}\sqrt{P_n}n_{i,k}\alpha_{i,k}^*\right|\right\} \to 1, \tag{4}$$

where $Pr\{\,\cdot\,\}$ represents the probability function.

**Lemma 1.** *$x \in R$ and $x > 0$, and $y$ is a complex number. If $x \gg |y|$, it has $|x + y| \approx x + \mathrm{Re}\{y\}$.*

Based on (3), (4), and Lemma 1, the envelope of MRC output is derived as

$$|r_i| = |r_i S^*| \approx \sum_{k=1}^{J}\sqrt{P_s}|\alpha_{i,k}|^2 + \mathrm{Re}\left\{\sum_{k=1}^{J}\sqrt{P_n}n_{i,k}\alpha_{i,k}^* S^*\right\}. \tag{5}$$

Using the approximation in (5), from (2), the envelope correlation between two MRC outputs is able to deduce as

$$\begin{aligned}\rho &= \frac{Cov[|r_1|, |r_2|]}{[Var(|r_1|)Var(|r_2|)]^{\frac{1}{2}}}\\ &= \frac{\sum_{k=1}^{J}P_s\beta_k\left[\sigma_{1,k}\sigma_{2,k}J_0(2\pi f_m\Delta\tau) + 2\pi\Delta f\tau_{rms}\bar{\varphi}_k + \varphi_k\right]}{\prod_{i=1}^{2}\left[\sum_{k=1}^{J}\left(\frac{P_n}{8}\left(2\sigma_{i,k}^2 + m_{i,k}^2 + \bar{m}_{i,k}^2\right) + P_s\sigma_{i,k}^2\left(\sigma_{i,k}^2 + m_{i,k}^2 + \bar{m}_{i,k}^2\right)\right)\right]^{\frac{1}{2}}},\end{aligned} \tag{6}$$

where $Var(\,\cdot\,)$ denotes the variance function, $\varphi_k = m_{1,k}m_{2,k} + \bar{m}_{1,k}\bar{m}_{2,k}$, and $\bar{\varphi}_k = \bar{m}_{1,k}m_{2,k} - m_{1,k}\bar{m}_{2,k}$.

Considering the identical distributed branches as in [4,5] that $m_{1,k} = m_{2,k} = m_k$, $\bar{m}_{1,k} = \bar{m}_{2,k} = \bar{m}_k$, $\Delta\tau = 0$ and $\sigma_{1,k} = \sigma_{2,k} = \sigma_k$, the envelope correlation in (6) is simplified as

$$\rho = \frac{2\gamma\xi(1 + 2K)\sum_{k=1}^{J}h_k^4}{(1 + K)^2 + 2\xi(1 + 2K)\sum_{k=1}^{J}h_k^4}, \tag{7}$$

where $\gamma = \left(1 + 4\pi^2\Delta f^2\tau_{rms}^2\right)^{-1}$, $\xi = P_s/P_n$ is the average SNR, $h_k = \sqrt{m_k^2 + \bar{m}_k^2 + 2\sigma_k^2}$ is the channel attenuation, and $K = (m_k^2 + \bar{m}_k^2)/2\sigma_k^2$ is the Rician factor.

### 2.2. Special cases

In this subsection, the properties of envelope correlation are investigated in several special cases.

#### 2.2.1. Noise neglect

If the effect of noise is eliminated, that is $\xi \to \infty$. Then, the envelope correlation in (7) is simplified as

$$\rho = \gamma. \tag{8}$$

From (8), we observe that the envelope correlation is directly determined by the frequency separation and the root mean square delay spread, which means that the effect of the multiple path can be ignored when the average SNR is extremely high. On the other hand, $\xi \to \infty$ indicates that $\xi \gg K$, which allows the effect of the direct path to be eliminated. Thus, the envelop correlation in the case of noise neglect can be simplified as the same as the single-branch case over the correlated Rayleigh fading channel [3].

#### 2.2.2. $K \gg \xi$

Consider the case $K \gg \xi$, we have

$$\rho = O\left(\frac{\xi}{K}\right)\gamma \ll \gamma, \tag{9}$$

which means that the envelope correlation will be reduced largely when the direct path introduces the major contribution to the received signal. This is reasonable because there is no envelope correlation between the direct paths.

#### 2.2.3. $\gamma \to 0$ or $\gamma \to 1$

When $\gamma \to 0$, we get $\rho \to 0$. While $\gamma \to 1$, we have

$$\rho = \frac{2\xi(1 + 2K)\sum_{k=1}^{J}h_k^4}{(1 + K)^2 + 2\xi(1 + 2K)\sum_{k=1}^{J}h_k^4}. \tag{10}$$

which indicates that the envelope correlation must be extremely small when $\gamma \to 0$, while it might be not large when $\gamma \to 1$.

Actually, in a real communication system, $\tau_{rms}$ is a given system parameter, which means that $\gamma \to 0$ equals to the case $\Delta f \to \infty$, while $\gamma \to 1$ equals to the case $\Delta f \to 0$. Thus, we can see that the envelop correlation can be reduced when the larger frequency separate is chosen.

## 3. The effect of envelope correlation

In this section, we investigate the effect of envelope correlation on system performance such as ergodic capacity and outage probability, and find out that some system parameters can be replaced by the envelope correlation to evaluate the system performance.