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The analysis of coupled interconnect with process parameter variation based wavelet collocation method

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ABSTRACT

In this paper, a novel approach based on fast wavelet collocation method (FWCM) is presented to solve partial differential equations (PDEs) in coupled on-chip interconnects with parameter variations. After processing PDEs by decomposing variables with wavelet functions, we transfer the PDEs into ordinary differential equations (ODEs), and then use Taylor expansion in the ODEs to approximate the partial complex expression containing inverse matrix. Consequently, we can solve the PDEs with random variables more feasibly. Moreover, this approach provides a new idea for solving other kinds of PDEs with random variables in very large scaled integrated circuits (VLSI). Comparison with HSPICE simulation results shows the method proposed in this paper is effective and accurate.

1. Introduction

As the VLSI fabrication technology stepped into the nanometer era, the sizes and intervals of interconnect on the chips shrink rapidly, and the System-on-Chip (SoC) signal frequency keeps increasing [1]. These trends in VLSI primarily affect parasitic resistance and capacitance of interconnects, further arise significant increase of crosstalk and the circuit delay, which have become critical in determining system performance and reliability. Crosstalk is a common phenomenon that often happens in the digital circuit design. It's caused by energy coupling between different interconnects. Mutual inductance and capacitance are the primary sources inducing crosstalk. There are two bad effects on circuits due to crosstalk: crosstalk will cause the change of effective characteristic impedance and propagation velocity, and affect systemic timing sequence; crosstalk will induce noise to the other interconnect wires, and reduce the noise tolerance and the signal integrity [2]. Circuit delay consists of electromagnetic wave transmission delay and rising edge delay. The delay determines the upper limit of the clock frequency, and it increases with the increasing of wire length. In the reality, because of the limitation of fabrication process, the distributed parameter of the interconnects cannot be constant, and even small variation of the distributed parameters will lead to great changes of circuit delay and crosstalk, so the analysis of the distributed parameter variation based on distributed parameter model is indispensable, many scholars and researchers work in this field in recent year [3–5].

Nowadays, there are various efficient simulation methods for timing analysis of the problem with random variations in the integrated circuits, and traditionally people use the computer simulation software like Pspice, that can conduct Monte-Carlo statistical analysis [6]. The core idea of Monte-Carlo in the electronic circuits is using a set of pseudo random numbers to obtain random sample sequences of the electric component parameters under the condition that the statistical distribution of given electric component parameter tolerances have been known, and then having DC, AC small signal and transient analysis on these random sample circuits for many times, followed by estimating the statistical distribution of the circuit performance. Although Monte-Carlo method is able to realistically describe the characteristics of random objects and physical experimental process, its convergence rate is comparatively slow to some numerical methods and its error is uncertain. In order to overcome this weakness, we choose the four-order B-spline FWCM [7] to deal with it. FWCM can handle circuit nonlinearity, control numerical accuracy, and will not cause the accumulation of numerical error since it works in time domain [8,9]. The wavelet property of localization in both the time and frequency domains makes a uniform approximation possible which is generally not found in time-marching methods [10]. Moreover, it has faster convergence rate than traditional algorithm. Compared with Monte-Carlo method, FWCM costs much less desired time to gain the similar precision.

In this paper, what we concern is the output voltage of the coupled interconnects and what we want to obtain is therelationship

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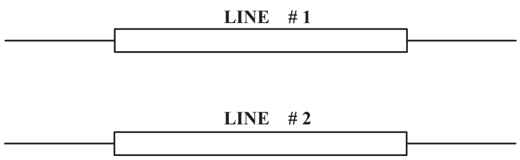

Fig. 1. Two coupled interconnects.

between the output voltage and the random distributed parameter variables. In Section 2, we establish the PDEs according to the distributed parameter model of the two coupled interconnects, then we transfer them into the ODEs based on four-order B-spline FWCM, finally we obtain the solutions (the output voltages) of PDEs and further derive their mean and variance. In Section 3, we compare our method with Monte-Carlo and demonstrate its effectiveness according to the simulation results. In Section 4, we conclude our study and comments on the future research.

2. Interconnect modeling and analysis

2.1. The establishment of PDEs in the coupled interconnects with random parasitic parameters

We study the two coupled interconnects shown in Fig. 1. The equivalent circuit diagram is shown in Fig. 2. Both of their lengths are l, and the variations of their distributed parameters are assumed to be random and normally distributed. We first normalize the length to L, which is also the solution interval. Suppose that R^*, L^*, G^* and C^* are the normalized resistance, inductance, conductance, and capacitance per unit length respectively.

It is easy to see that R^*, L^*, G^* and C^* are composed of two parts:

$$\begin{aligned} R^* &= R + \Delta R, \quad L^* = L + \Delta L, \\ G^* &= G + \Delta G, \quad C^* = C + \Delta C. \end{aligned} \tag{1}$$

where

$$R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \Omega/\text{m}, \quad L = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \text{nH/m},$$

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \text{pF/m}, \quad G = 0;$$

$$\Delta R = \begin{bmatrix} 0.1 * R_{11} * X & 0 \\ 0 & 0.1 * R_{22} * X \end{bmatrix} \Omega/\text{m},$$

$$\Delta L = \begin{bmatrix} 0.1 * L_{11} * Y & 0.1 * L_{12} * Z \\ 0.1 * L_{21} * Z & 0.1 * L_{22} * Y \end{bmatrix} \text{nH/m},$$

$$\Delta C = \begin{bmatrix} 0.1 * C_{11} * W + 0.1 * C_{12} * Q & 0.1 * C_{12} * Q \\ 0.1 * C_{21} * Q & 0.1 * C_{22} * W + 0.1 * C_{21} * Q \end{bmatrix} \times \text{pF/m}, \quad \Delta G = 0.$$

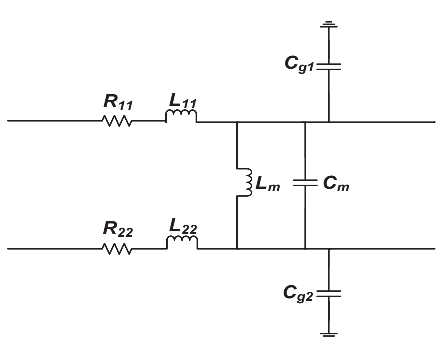

Fig. 2. Equivalent circuit diagram of the two coupled interconnects.

without loss of generality, we assume the maximum variation of electrical parameters is 10% (0.1) of normal value. R_{11} is the self-resistance of line 1, R_{22} is the self-resistance of line 2, and R_{12} and R_{21} are mutual resistances, $R_{12} = R_{21} = R_m$, and they do not have important impact on the electrical performance of interconnects, so here we let $R_m = 0$; L_{11} is the self-inductance of line 1, L_{22} is the self-inductance of line 2, L_{12} is the mutual inductance between line 1 and line 2, L_{21} is the mutual inductance of between line 2 and line 1, $L_{12} = L_{21} = L_m$; C_{12} is the mutual capacitance between line 1 and line 2, C_{21} is the mutual capacitance between line 2 and line 1, $C_{12} = C_{21} = C_m$, C_{11} is the sum of the ground capacitance C_{g1} of line 1 and C_{12}, C_{22} is the sum of the ground capacitance C_{g2} of line 2 and C_{21}, i.e., $C_{11} = C_{g1} + C_{12}$, $C_{22} = C_{g2} + C_{21}$; and we suppose G is 0, it is true for most of insulation materials; X, Y, Z, W, Q all obey standard normal distribution, i.e. $X, Y, Z, W, Q–N(0,1)$, and $X, Y, Z, W, Q \in (-1, 1)$.

Let z be the axis along the line, and $z = 0$ and $z = L$ correspond to its near and far end respectively. Further, let $v(z, t)$ and $i(z, t)$ be the voltage and current along the line. The PDEs of the line can be derived from Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL), written as follows [11]:

$$\frac{\partial v(z,t)}{\partial z} = -R^*(z)i(z,t) - L^*(z)\frac{\partial i(z,t)}{\partial t} \tag{2}$$

$$\frac{\partial i(z,t)}{\partial z} = -G^*(z)v(z,t) - C^*(z)\frac{\partial v(z,t)}{\partial t} \tag{3}$$

With the boundary conditions:

$$V(0,t) = V_{bl}(t), \quad V(L,t) = V_{br}(t) \tag{4}$$

For Laplace transforms on the above PDEs, we can obtain:

$$\frac{dV(z,s)}{\partial z} = -(R^*(z) + SL^*(z))I(z,s) \tag{5}$$

$$\frac{dI(z,s)}{\partial z} = -(G^*(z) + SC^*(z))V(z,s) \tag{6}$$

Let

$$U(z,s) = \begin{bmatrix} I(z,s) \\ V(z,s) \end{bmatrix}, \quad T = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$O = \begin{bmatrix} R^*(z) & 0 \\ 0 & G^*(z) \end{bmatrix}, M = \begin{bmatrix} L^*(z) & 0 \\ 0 & C^*(z) \end{bmatrix}.$$

Then we rewrite the PDEs in the following matrix form:

$$\left(T\frac{d}{dz} + O + SM\right)U(z,s) = 0 \tag{7}$$

2.2. The introduction of the four-order B-spline wavelet function

Let $H^2(I)$ be the Sobolev space which basically contains functions equipped with a norm that is a combination of L^p-norms of the function itself as well as its derivatives up to second order [12]. Let I denote a standard interval, say $I = [0, L]$, then we introduce approximation subspace $V_{bj} \subset H^2(I)$ for a given integer number $J \geq 0$, consisting of scaling functions and wavelet functions.