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Short communication

Improving satellite image classification by using fractional type convolution filtering

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ABSTRACT

This letter shows how conventional methods for satellite image classification can be improved by applying some filtering algorithms as a pre-classifying step. We will introduce a filtering scheme based on convolution equations of fractional type. The use of this kind of filter as a pre-classification step will be illustrated by classifying MODerate-resolution Imaging Spectroradiometer (MODIS) data to map burned areas in Mediterranean countries. The methodology we propose improved the estimations obtained by merely classifying the post-fire images (i.e. without filtering) in the study areas considered.

1. Introduction

We present an approach to image classification including, as the main novelty, a pre-classifier based on fractional type convolution equations which has been show highly efficient in previous implementations (Cuesta and Finat, 2003). To illustrate the improvements achieved by our methodology we will map some areas burned by forest fires by adopting a single-data approach.

As a first attempt, we will simply use original bands of the post-fire image (without any transformation) as inputs of a conventional classifier. Nor vegetation indexes neither any other synthetic bands will be used, though they had been shown to produce more accurate estimations (e.g. Pereira, 1999; Quintano et al., 2006; Quintano and Shimabukuro, 2009). Let us notice that similar improvements are expected by classifying synthetic bands and/or by using more sophisticated classification algorithms.

The new features of the proposed procedure can be sketched as:

- The fractional filter we propose as pre-classifier fits into a closed mathematical framework of well-posedness and numerical solvability, on the contrary what happens to the classical non-linear partial differential equations based approaches.
- This filter increases the intra-class spectral variability and reduce the inter-class spectral differences which lead to improve each simple classification as several authors recommended (see, e.g. Tottrup, 2004; Quintano et al., in press).

2. Convolution type equations and convolution type numerical methods

In the framework of partial differential equations, the linear heat equation:

$$\begin{cases} \partial_t u(t, \mathbf{x}) = \Delta u(t, \mathbf{x}), & (t, \mathbf{x}) \in [0, T] \times \Omega, \\ u(0, \mathbf{x}) = u_0(\mathbf{x}), & \mathbf{x} \in \Omega, \end{cases} \tag{1}$$

where ∂_t stands for the time derivative, and $\Delta = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$ is defined in a domain $\Omega \subset \mathbb{R}^2$ (typically a square domain) with homogeneous boundary conditions, has been shown to be the simplest tool for image processing (filtering, de-noising, segmentation, restoration, etc.). However, Eq. (1) yields usually filtered images which in practice turn out to be unacceptable, mainly because the edges and vertices are severely blurred. To overcome this draw-

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back non-linear approaches have been studied in literature, for example those based on replacing Δu by an anisotropic diffusion $\nabla(d(|\nabla u|)\nabla u)$ where the diffusion is then handled by $d(|\nabla u|)$. The diffusion preserves edges and vertices if $d : [0, +\infty) \to [0, +\infty)$ satisfies

- $d(s)$ decreases as $s \to +\infty$.
- $d(0) = 1$ and $d(s) \approx 1/\sqrt{s}$, as $s \to +\infty$.
- $d(s) + 3sd'(s) > 0$ (parabolic condition).

Unfortunately most choices of d lead to ill-posed initial-boundary value problems (see Aubert and Kornprobst, 2000, for further details, examples, and references). For instance, in Velasco-Forero and Manian (2009) a pre-classifier based on a suitable choice of d is used. However, the authors fail to mention either the well-posedness of the problem or the validity of the discretization proposed for such a problem.

Image filtering based on fractional calculus was first proposed in Cuesta and Finat (2003) and later studied by other authors (see, e.g. Bai and Feng, 2007). In particular, the procedure described in Cuesta and Finat (2003) makes use of the partial fractional differential equation:

$$\begin{cases} \partial_t^{\alpha} u(t, \mathbf{x}) = \Delta u(t, \mathbf{x}), & (t, \mathbf{x}) \in [0, T] \times \Omega, \\ u(0, \mathbf{x}) = u_0(\mathbf{x}), & \end{cases} \quad (2)$$

where ∂_t^{α} stands for the time derivative of order $\alpha > 0$ (even for non-integer α) in the sense of Riemann–Liouville (see Podlubny, 1999), and u_0 stands for the original image. The main goal of this approach is that the diffusion (i.e. the smoothing effect) is now handled by means of the parameter α, i.e. without considering additional non-linearities, as commented above for Eq. (1). This property is a consequence of the fact that, for $1 < \alpha < 2$, Eq. (2) interpolates the heat Eq. (1) and the wave equation (with zero initial velocity):

$$\begin{cases} \partial_t^{2} u(t, \mathbf{x}) = \Delta u(t, \mathbf{x}), & (t, \mathbf{x}) \in [0, T] \times \Omega, \\ u(0, \mathbf{x}) = u_0(\mathbf{x}), & \partial_t^{1} u(0, \mathbf{x}) = 0, \end{cases} \quad (3)$$

$\alpha = 1$ and $\alpha = 2$, respectively. It has been proven that the solution of (2) satisfies certain intermediate properties between (1) and (3), in particular the diffusion decreases as α goes from 1 to 2. On the other hand, the well-posedness of problem (2) is very well known, and therefore, this fact is not a gap in our methodology.

By integrating with order α in both sides, we can re-write Eq. (2) in integral form as

$$u(t, \mathbf{x}) = u_0(\mathbf{x}) + \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \Delta u(s, \mathbf{x}) \, ds, \quad (4)$$

Numerical discretizations of Eq. (4) have been studied in literature (see, e.g. Cuesta et al., 2008, and references therein). In fact, the convolution quadrature based on the backward Euler method is used for the time discretization (see Cuesta and Palencia, 2003), and a second-order finite differences method for the Laplacian. To be more precise, let $\mathbf{U}_0$ be the vector whose components stand for the gray levels of an $M \times M$ gray-scale image, $\tau > 0$ a time step length, $\mathbf{U}_n$ the numerical solution at time level $t_n = n\tau$ and let Δ_h be the $M^2 \times M^2$ matrix corresponding to the discretization of the Laplacian, as mentioned above. Hence, the fully discrete equation reads:

$$\mathbf{U}_n = \mathbf{U}_0 + \sum_{j=0}^{n} q_{n-j}^{(\alpha)} \Delta_h \mathbf{U}_j, \quad 1 \le n \le N. \quad (5)$$

Let us recall that the weights $q_j^{(\alpha)}$, $j = 1, 2, \ldots$ of the convolution quadrature based on the backward Euler method are defined as the coefficients of the power expansion of the generating function $(\tau/(z-1))^{\alpha}$ which can be computed as

$$q_j^{(\alpha)} = \tau^{\alpha} \binom{\alpha}{j}, \quad j \ge 0. \quad (6)$$

In practice, an efficient computation of the weights can be carried out by means of the fast Fourier transform (FFT) as we did.

In this work, a finer approach is applied which yields better practical results. Such an approach consists of splitting the whole image into sub-images, and then applying the numerical method (5)–(6) with different choices of α on each sub-image (see Cuesta, in press). In particular, the parameter α is chosen between 1 and 2 by taking the value closer to 2 for the sub-images with higher mean gradient variation (i.e. with "a lot" of edges, vertices, spots, etc.), and the value closer to 1 for the ones with lower mean gradient variation (i.e. "few" edges, vertices, etc.). Let us note that the measure of the gradient is not an issue in this paper and, therefore, we simply apply a discrete measure which becomes natural in the framework of digital images.

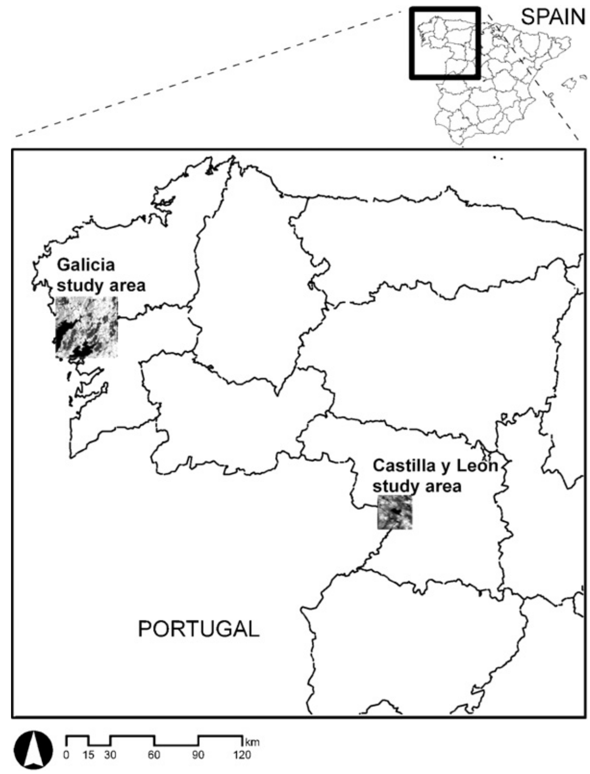

Fig. 1. Study areas location

3. Materials and methods

3.1. Study area and dataset

Two Spanish regions were considered as study areas (Fig. 1). On the one hand, Castilla y León, located in the central northern area of Spain, where a large forest fire occurred on 17 July, 2004 in the region of Fonfría (latitude 41.722–41.518°, longitude −6.299° to −6.007°). On the other hand Galicia, located in the northwest of the country, where several forest fires took place from 4 August up to 15 August, 2006 in the Barbanza district (latitude 42.943–42.570°, longitude −9.120° to −8.559°).

For our experiments we used the data from three different sources: