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A diffusion approximation model for wireless networks based on IEEE 802.11 standard

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ABSTRACT

The article presents an analytical model of wireless networks using the IEEE 802.11 protocol to access the transport medium. The model allows to determine such key factors of the quality of service as transmission delays and losses. The model is based on diffusion approximation approach which was proposed three decades ago to model wired networks. We show that it can be adapted to take into consideration the input streams with general interarrival time distributions and servers with general service time distributions. The diffusion approximation has been chosen because of fairly general assumptions of models based on it, hard to be represented in Markov models. A queueing network model can have an arbitrary topology, the intensity of transmitted flows can be represented by non-Poisson (even self-similar) streams, the service times at nodes can be defined by general distributions. These assumptions are important: because of the CSMA/CA algorithm, the overall times needed to sent a packet are far from being exponentially distributed and therefore the flows between nodes are non-Poisson. Diffusion approximation allows us also to analyse the of transient behaviour of a network when traffic intensity is changing with time.

1. Introduction

The traffic transmitted by wireless networks has become more and more important, hence the performance and QoS issues of these networks should be carefully studied. The performance of IEEE 802.11 standard for wireless networks, its Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) scheme with exponential backoff mechanism and its variants used to support asynchronous transfers, were thoroughly studied either analytically or by simulation e.g. in [1,15,4,2,21,25,26]. The studies usually refer to the limit high traffic conditions. The relationships among throughput, blocking and collision probabilities are obtained, often with the use of a discrete-time Markov chain and then the performance of the backoff mechanism is studied.

Here, we propose a model which allows us to study not only the throughput of nodes using IEEE 802.11 standard, but also predicts the queue distributions at each node of the studied network, as well as waiting times distributions – hence the-end-to-end delays – and the loss probabilities due to the buffer overflows.

The model is based on the diffusion approximation which is a classical modelling method developed in 70-ties [12–14] to study the performance of wired networks. The model is a typical queueing network one, where service stations represent nodes, service times represent the time needed to send a packet and queues at stations model the queues of packets at nodes. Once we obtain the queue distribution, we may also predict the waiting time distribution and the probability that the queue reaches its maximum value which approximates packet loss probability due to a saturated buffer.

The method can be used to model networks composed of a large number of nodes, e.g. mesh networks. It also allows the analysis of transient states occurring because of time-dependent traffic intensity or because of the changes in the network topology.

The main contribution of the article is a discussion how the CSMA/CA scheme with exponential backoff mechanism can be incorporated in this model and showing how the new model can be solved numerically.

The article is organised as follows. Section 2 recalls the principles of standard diffusion approximation model applied to a single station with limited queues with general independent distributions of interarrival and service times, i.e. the G/G/1/N queue. Both steady state and transient state solutions are given. The transient state model was proposed previously by the authors, [6,7]. Section 3 presents the diffusion approximation model of a single node using CSMA/CA scheme with exponential backoff mechanism. Section 4 shows how the entire network of nodes with arbitrary topology can be solved. Some numerical examples are given.

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2. Diffusion model of a G/G/1/N station

Let $A(x), B(x)$ denote the interarrival and service time distributions at a service station. The distributions are general, but not specified, and the method requires only their first two moments. The means are $E[A] = 1/\lambda, E[B] = 1/\mu$ and variances are $\text{Var}[A] = \sigma_A^2, \text{Var}[B] = \sigma_B^2$. We also denote squared coefficients of variation as $C_A^2 = \sigma_A^2\lambda^2$ and $C_B^2 = \sigma_B^2\mu^2 \cdot N(t)$ represents the number of customers present in the system at time t.

As we assume that the interarrival times are independent and identically distributed random variables, hence, according to the central limit theorem, the number of customers arriving at the interval of length t (sufficiently long to ensure a large number of arrivals) can be approximated by the normal distribution with mean λt and variance $\sigma_A^2\lambda^3 t$. Similarly, the number of customers served in this time is approximately normally distributed with mean μt and variance $\sigma_B^2\mu^3 t$, provided that the server is busy all the time. Consequently, the changes of $N(t)$ within interval $[0,t]$, $N(t) - N(0)$, have approximately normal distribution with mean $(\lambda - \mu)t$ and variance $(\sigma_A^2\lambda^3 + \sigma_B^2\mu^3)t$.

Diffusion approximation [22,23] replaces the process $N(t)$ by a continuous diffusion process $X(t)$. The incremental changes of $X(t)$, $dX(t) = X(t + dt) - X(t)$ are normally distributed with the mean βdt and variance αdt, where β, α are the coefficients of the diffusion equation

$$\frac{\partial f(x,t;x_0)}{\partial t} = \frac{\alpha}{2}\frac{\partial^2 f(x,t;x_0)}{\partial x^2} - \beta\frac{\partial f(x,t;x_0)}{\partial x}, \tag{1}$$

which defines the conditional pdf of $X(t)$

$$f(x,t;x_0) = P[x \leqslant X(t) < x + dx | X(0) = x_0].$$

Both processes $X(t)$ and $N(t)$ have normally distributed changes; the choice $\beta = \lambda - \mu, \alpha = \sigma_A^2\lambda^3 + \sigma_B^2\mu^3 = C_A^2\lambda + C_B^2\mu$ ensures the same ratio of time-growth of mean and variance of these distributions. The density of the diffusion process approximates the distribution of $N(t)$: $p(n,t;n_0) \approx f(n,t;n_0)$, and in steady state $p(n) \approx f(n)$.

More formal justification of diffusion approximation is in limit theorems for G/G/1 system given by Iglehart and Whitt [16,17], but only for nonstationary processes.

The process $N(t)$ is never negative, hence $X(t)$ should be also restrained to $x \geqslant 0$. A simple solution is to put a *reflecting barrier* at $x = 0$ [19,20].

The reflecting barrier excludes the stay at zero: the process is immediately reflected, therefore this version of diffusion with reflecting barrier is a heavy-load approximation. This inconvenience can be removed by the introduction of another limit condition at $x = 0$: *a barrier with instantaneous (elementary) jumps* [12]. When the diffusion process comes to $x = 0$, it remains there for the time exponentially distributed with a parameter λ_0 and then it returns to $x = 1$. The time when the process is at $x = 0$ corresponds to the idle time of the system.

In the case of a queue limited to N positions, the second barrier of the same type is placed at $x = N$. Coming to the barrier at $x = N$, the process stays there for a time corresponding to the period when the queue is full and incoming customers are lost and then, after the completion of the current service, the process jumps to $x = N - 1$. The model equations become [12]

$$\begin{aligned}
\frac{\partial f(x,t;x_0)}{\partial t} &= \frac{\alpha}{2}\frac{\partial^2 f(x,t;x_0)}{\partial x^2} - \beta\frac{\partial f(x,t;x_0)}{\partial x} + \\
&\quad \lambda_0 p_0(t)\delta(x-1) + \lambda_N p_N(t)\delta(x - N + 1),\\
\frac{dp_0(t)}{dt} &= \lim_{x\to 0}\left[\frac{\alpha}{2}\frac{\partial f(x,t;x_0)}{\partial x} - \beta f(x,t;x_0)\right] - \lambda_0 p_0(t),\\
\frac{dp_N(t)}{dt} &= \lim_{x\to N}\left[-\frac{\alpha}{2}\frac{\partial f(x,t;x_0)}{\partial x} + \beta f(x,t;x_0)\right] - \lambda_N p_N(t),
\end{aligned} \tag{2}$$

where $\delta(x)$ is Dirac delta function.

2.1. Steady state

In stationary state Eq. (2) become ordinary differential ones and their solution, if $\varrho = \lambda/\mu \neq 1$, can be expressed as:

$$f(x) = \begin{cases} \frac{\lambda p_0}{-\beta}(1 - e^{zx}), & \text{for} \quad 0 < x \leqslant 1,\\ \frac{\lambda p_0}{-\beta}(e^{-z} - 1)e^{zx}, & \text{for} \quad 1 \leqslant x \leqslant N - 1,\\ \frac{\mu p_N}{-\beta}(e^{z(x-N)} - 1), & \text{for} \quad N - 1 \leqslant x < N, \end{cases}$$

where $z = \frac{2\beta}{\alpha}$ and p_0, p_N are determined through normalization

$$p_0 = \lim_{t\to\infty} p_0(t) = \left\{1 + \varrho e^{z(N-1)} + \frac{\varrho}{1-\varrho}[1 - e^{z(N-1)}]\right\}^{-1},$$
$$p_N = \lim_{t\to\infty} p_N(t) = \varrho p_0 e^{z(N-1)}.$$

The steady state solution does not depend on the distributions of the sojourn times in boundaries, but only on their first moments.

2.2. Transient state

First, we obtain the density $\phi(x,t;x_0)$ of the diffusion process with two absorbing barriers at $x = 0$ and $x = N$, started at $t = 0$ from $x = x_0$, cf. [5]

$$\phi(x,t;x_0) = \frac{1}{\sqrt{2\Pi\alpha t}}\sum_{n=-\infty}^{\infty}(a_n - b_n),$$

where

$$a_n = \exp\left[\frac{\beta x'_n}{\alpha} - \frac{(x - x_0 - x'_n - \beta t)^2}{2\alpha t}\right],$$
$$b_n = \exp\left[\frac{\beta x''_n}{\alpha} - \frac{(x - x_0 - x''_n - \beta t)^2}{2\alpha t}\right],$$

and $x'_n = 2nN, x''_n = -2x_0 - x'_n$.

If the initial condition is defined by a function $\psi(x)$, $x \in (0,N)$, $\lim_{x\to 0}\psi(x) = \lim_{x\to N}\psi(x) = 0$, then the pdf of the process has the form $\phi(x,t;\psi) = \int_0^N \phi(x,t;\xi)\psi(\xi)d\xi$.

Then the pdf $f(x,t;\psi)$ of the diffusion process with elementary returns from both barriers is expressed as

$$\begin{aligned} f(x,t;\psi) = \phi(x,t;\psi) &+ \int_0^t g_1(\tau)\phi(x,t-\tau;1)d\tau \\ &+ \int_0^t g_{N-1}(\tau)\phi(x,t-\tau;N-1)d\tau. \end{aligned} \tag{3}$$

The densities $g_1(t)$ and $g_N(t)$ of starting new processes at $x = 1$ and $x = N - 1$ due to jumps from neighboring barriers can be defined with the use of functions $\gamma_0(t)$ and $\gamma_N(t)$:

$$g_1(\tau) = \int_0^\tau \gamma_0(t) l_0(\tau - t)dt,$$
$$g_{N-1}(\tau) = \int_0^\tau \gamma_N(t) l_N(\tau - t)dt,$$

where $l_0(x), l_N(x)$ are the densities of sojourn times in $x = 0$ and $x = N$. Note that the distributions of these times are not restricted to exponential ones. The densities $\gamma_0(t), \gamma_N(t)$ of the probability that at time t the process enters to $x = 0$ or $x = N$ depend in turn on $g_1(t)$ and $g_N(t)$:

$$\begin{aligned} \gamma_0(t) = p_0(0)\delta(t) + [1 - p_0(0) - p_N(0)]\gamma_{\psi,0}(t) \\ + \int_0^t g_1(\tau)\gamma_{1,0}(t-\tau)d\tau \\ + \int_0^t g_{N-1}(\tau)\gamma_{N-1,0}(t-\tau)d\tau, \end{aligned}$$