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A multi-component two-phase flow algorithm for use in landfill processes modelling

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ABSTRACT

This paper describes the finite difference algorithm that has been developed for the flow sub-model of the University of Southampton landfill degradation and transport model LDAT. The liquid and gas phase flow components are first decoupled from the solid phase of the full multi-phase, multi-component landfill process constitutive equations and are then rearranged into a format that can be applied as a calculation procedure within the framework of a three dimensional array of finite difference rectangular elements.

The algorithm contains a source term which accommodates the non-flow landfill processes of degradation, gas solubility, and leachate chemical equilibrium, sub-models that have been described in White and Beaven (2013).

The paper includes an illustration of the application of the flow sub-model in the context of the leachate recirculation tests carried out at the Beddington landfill project. This illustration demonstrates the ability of the sub-model to track movement in the gas phase as well as the liquid phase, and to simulate multi-directional flow patterns that are different in each of the phases.

1. Introduction

Multi-phase multi-component landfill processes models have a role in supporting the design of landfill management and engineering systems that implement leachate flushing/recirculation, aeration, and gas extraction and collection. There is a requirement for such models to have the capability to represent the multi-component two phase flow of liquid and gas through the pore space of the solid phase of the waste material.

This paper describes the liquid and gas flow algorithm in the University of Southampton landfill degradation and transport model LDAT. The algorithm has been significantly refined since it was first published, (White et al., 2004). Since 2004 the following new developments have been added to the flow sub-model:

- The gas and liquid phases are now treated separately. They each have their own pressure fields which are linked using a van Genuchten capillary pressure function, Eq. (16). This permits the liquid and gas dynamics to be modelled more realistically and accommodates the drainage behaviour of different landfill materials under varying degrees of compaction and degradation.
- Multi-component gas and liquid flow has been incorporated into the model allowing for the determination of the detailed composition of gas and liquid fluxes both at the boundaries of the model and within the body of the model. These compositions are used by the chemical equilibrium sub-model to determine the leachate chemistry, (White and Beaven, 2013).
- The concept of effective density has been developed for both phases to account for entrapments of liquid around a gas phase and vice versa, Eqs. (12) and (13). This is used in conjunction with the van Genuchten functions for the relative gas and liquid permeabilities to provide a new constitutive equation based on a modified Darcy's Law, constructed using Eqs. (3)–(10).

LDAT is a general model of landfill processes consisting of a set of linked process sub-models. Each process sub-model references the same set of Primary Variables, which are sufficient to specify the state of the waste at any point in time and space. All other variables, the Secondary Variables, required by a sub-model are derived from the Primary Variables using a set of common constants and empirical relationships. Examples of Secondary Variables include density and permeability.

In the case of LDAT the Primary Variables are the mass of each chemical compound in each of the solid, liquid and gas phases m_n^p, together with the stresses applied to those phases. The stresses are the total stress applied to the solid phase, σ, the liquid and gas

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Nomenclature

C	subscript denoting capillary pressure see Eq. (16)
E	subscript denoting 'effective'
e	subscript indexing finite difference element see Fig. 1
m	subscript indexing elements neighbouring *e*, see Fig. 1
n	subscript indexing component
REF	subscript denoting reference value
REL	subscript denoting relative value
vh	subscript denoting vertical to horizontal ratio
D	superscript denoting dry density see Eq. (17)
G	superscript denoting gas phase
L	superscript denoting liquid phase
MAX	superscript denoting maximum value
MIN	superscript denoting minimum value
P	superscript denoting phase. Liquid phase P = L, gas phase P = L, solid phase P = S
R	superscript denoting residual value
S	superscript denoting solid phase
A	area m^2
a	see Eq. (27) m^3/kPa
B	right hand side of solution matrix, see Eq. (34) m^3
b	see Eqs. (28) and (29) m^3
C	left hand side of solution matrix, see Eq. (34) m^3/kPa
c	see Eq. (31) m^3
d	element dimension m
G	source flow concentration $m^3/(day.m^3)$
g	acceleration due to gravity kN/kg
h	pressure head m
i	column index in solution matrix
J	mass transmissivity, see Eq. (20) kg/(day.m)
k, K	permeability m/day
l	absolute value of distance between element centres m
M	mass transfer kg
m	mass kg
p	pressure kPa
Q	volume transfer m^3
S	storage coefficient 1/kPa
S′	volumetric storage coefficient, see Eq. (31) m^3/kPa
T	temperature °C, K
t	time day
V	volume m^3
v	component flow velocity m/day
x	coordinate m
z	concentration
α	see Eq. (16) m^3/m^3
β	see Eq. (16) m^{-1}
δ	see Eqs. (8) and (9)
ε	see Eqs. (14) and (15)
ϕ	porosity
γ	see Eqs. (8), (9), and (16)
η	see Eq. (5)
λ	see Eq. (17)
μ	viscosity kPa. day
θ	moisture content
ρ	density kg/m^3
σ	total stress kPa
σ_0	surface stress kPa
σ'	effective stress kPa
ς	degree of saturation
τ	see Eq. (15)
ω	see Eq. (22) m^2

pressures, p^L and p^G, and temperature *T*. Further details of the sub-model structure and the various variables and constants that are employed by the model are given in White and Beaven (2013).

The sub-models together effectively solve the constitutive equations of landfill waste degradation and transport within the framework of a finite difference grid that represents the landfill spatially.

The source term of the constitutive equation is arranged to accommodate degradation, gas solubility, gas diffusion, and the impact of temperature changes due to heat generation and transfer. The remainder of the equation is decoupled from the solid phase terms which form the basis of the settlement sub-model, with liquid/gas phase transport calculated in a sub-model as described in Sections 2 and 3.

Section 4 illustrates the application of the liquid/gas sub-model in the context of the leachate recirculation tests carried out at Beddington Farmlands landfill project, and demonstrates the ability of the sub-model to track the gas phase as well as the liquid phase.

Following the description of the LDAT degradation algorithm given in White and Beaven (2013), the aim of this paper is to provide details of the LDAT flow algorithm and to demonstrate its capability with a conceptual application simulating leachate recirculation.

2. The LDAT liquid and gas flow constitutive equations

The bio-chemical and physical processes that take place in a waste landfill may be represented analytically by a set of algebraic constitutive equations. The solution of these equations may be obtained numerically by re-casting them as a calculation procedure, or algorithm. This section describes the constitutive equations and algorithm that may be used to represent the flow of the liquid and gas phases in the waste material. These are based on those used in the landfill degradation and transport model LDAT (White et al., 2004; White and Beaven, 2013) which in turn have been derived from a range of sources including (Darcy, 1856; Bear, 1979; Das, 1983; McDonald and Harbaugh, 1988; Ghabaee and Rodwell, 1989; Bear and Verruijt, 1992; Moody et al., 1992; Hydrogeologic, 1996; Droste, 1997; Bente, 2011).

LDAT solves the landfill process constitutive equations using a finite difference algorithm within a framework of rectangular representative elementary volumes. The framework for the finite difference algorithm is a three dimensional rectangular element *e* as shown in Fig. 1.

The space occupied by the waste material in the landfill is represented by a three dimensional array of these elements. Each element is surrounded by six neighbouring elements some of which may be boundary elements. The array of six neighbouring elements is indexed by $m = 1–6$, Fig. 1. The suffix *em* attached to a variable denotes that the variable is evaluated at the interface between the element and its neighbour *m*. In the case of vectors it also denotes direction, which is positive in the outward direction normal to the interface. A single suffix *e* attached to a variable denotes that the variable is evaluated at the centroid of element *e*.

The waste material is represented as the assembly of a number of component chemical compounds and species (chemical elements), each of which can exist in one or all of the three phases solid, liquid, and gas. The conservation of the mass $m^P_{e,n} = \rho^P_{e,n} z^P_{e,n} V_e$ of the *n*th component of the waste in phase *P* (solid, liquid or gas) in the context of a representative elementary volume, V_e, may be expressed by the following equation,