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Boost image denoising via noise level estimation in quaternion wavelet domain

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ABSTRACT

Wavelet thresholding is an important branch of the image denoising field. A key parameter in the algorithms is noise level. As a novel tool of image analysis, quaternion wavelet owns some superior properties compared to discrete wavelets, such as nearly shift-invariant wavelet coefficients and phase based texture presentation. We aim to propose an easy and efficient method to estimate the noise level accurately via quaternion wavelet and further improve the denoising performance. We find that the variance sum of high frequency coefficients of quaternion wavelet is approximately equal to the noise level. However, with the advent of strong edges and/or less smooth regions, this metric would overestimate the noise level. Phases in the quaternion wavelet domain can represent the image texture information. On the premise of detecting smooth regions via phases operation, i.e. without many textures, the proposed noise level estimation method is also suitable to images with complex scenes. The performance of the proposed noise level estimation algorithm is demonstrated superior to classical algorithms. Also, the proposed algorithm can enhance those noise level dependent techniques to improve the denoising performances which are competitive to the state-of-art denoising algorithms.

1. Introduction

Research on image denoising has been lasting for decades in the field of computer vision, image processing, and digital imaging etc. The most representative denoising methods are BM3D [1], BLS-GSM [2], NLM [3]. BM3D groups overlapping blocks to achieve sparsity enhancement and feature preservation via the collaborative filtering in transform-domain. The BLS-GSM is based on the rigorous statistical criteria and multi-resolution image analysis. However, they both need to calibrate the noise parameters in order to receive optimal performance. As many other literatures assume additive white Gaussian noise for natural images, the noise level is given as the input parameter for denoising algorithms [1], especially for wavelet based methods [2,4–6]. However, this is not the case in practical use. Thus, most of the research papers [7–12] regard the median value of the wavelet coefficients at the finest decomposition level (subband HH_1) as the noise level [13] while this often leads to significant overestimates [14]. The advantage of [15] is that it includes a Laplacian operation which is almost insensitive to image structure but only depends on the noise in the image, but the estimation result is also not accurate.

As a novel image analysis tool, quaternion wavelet transform (QWT) owns some superior properties compared to discrete wavelets, such as nearly shift-invariant wavelet coefficients and the ability of texture presentation which provides richer image texture information because of the phases. Chan et al. [16] extend the idea of dual tree complex wavelet to the quaternion domain using the concepts of 2D Hilbert transform and analytic signal with the application to disparity estimation. Until now, QWT has been applied to image denoising [10,17,18], texture classification [19–21], global and local blur degree detection [22,23] and image fusion [24,25]. QWT based image denoising exhibits better performance, although it mainly extends wavelet algorithms into quaternion wavelet domain. QWT provides quaternion valued subbands with the nearly shift-invariant magnitude and three 2D phases containing geometric information with a coherent description of local 2D structure, so phase preserving denoising has shown great potential. In spite of this, we expect to further boost the denoising performance by means of noise level estimation, because Bayeshrink thresholding based quaternion wavelet denoising in [18] still exploits the noise level estimation method like [13].

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In this paper, we focus on noise level estimation via quaternion wavelet rather than common methods in spatial domain, because computational load is reduced if both the noise level estimation and denoising method are based on quaternion wavelet. Quaternion wavelet domain denoising performance is enhanced via the noise level estimation. We use QWT to transform natural images into the quaternion magnitude-phase domain, and compute the variance sum of the high frequency coefficients. In our finding, the computation value can approximately represent the noise level. It is a global index, called '**NLE1**', in case that noise level estimation is overestimated, we further propose one universal metric '**NLE2**' considering local smooth regions detection that helps to improve the quality of the denoised image from the perspectives of both the visual perception and performance evaluation index.

The rest of this paper is organized as follows. The basic knowledge about quaternion and construction of quaternion wavelet is illustrated in Section 2. In Section 3, the proposed noise level estimation algorithm is presented. The experimental results are given, and the proposed noise level estimation and the following denoising performances are also discussed in contrast to the existing algorithms in Section 4. Finally, we draw the conclusions in Section 5.

2. Quaternion wavelet transform

Quaternion wavelet transform (QWT) is the extension of complex wavelet transform that provides a richer scale-space analysis for 2-D signals' geometric structure. Contrary to discrete wavelet transform (DWT), it is near shift invariant and provides a magnitude-phases local analysis of images. For convenience of further discussions, we briefly review some basic ideas on quaternion and construction of QWT.

2.1. Basic concepts of quaternion

The quaternion algebra $\mathcal{H}$ was invented by Hamilton in 1843 which is a generalization of the complex algebra.

$$\mathcal{H} = \{q = a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k} | a, b, c, d \in \mathbf{R}\} \quad (1)$$

where the orthogonal imaginary numbers $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ satisfy the following rules

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1, \quad \mathbf{ij} = \mathbf{k}, \quad \mathbf{jk} = \mathbf{i}, \quad \mathbf{ki} = \mathbf{j} \quad (2)$$

An alternative representation for a quaternion is

$$q = |q| e^{i\phi} e^{k\psi} e^{j\theta} \quad (3)$$

where $(\phi, \theta, \psi) \in [-\pi, \pi) \times \left[-\pi/2, \pi/2\right) \times \left[-\pi/4, \pi/4\right]$. It is defined by one magnitude and three angles that we call phase. When $\psi \in (-\pi/4, -\pi/4)$, The computational formulae [26] is

$$\begin{cases} \phi = \arctan\left(\dfrac{2(ac+bd)}{a^2+b^2-c^2-d^2}\right) \\ \theta = \arctan\left(\dfrac{2(ab+cd)}{a^2-b^2+c^2-d^2}\right) \\ \psi = -\dfrac{1}{2}\arcsin\left(2(bc-ad)\right) \end{cases} \quad (4)$$

2.2. Quaternion wavelet transform

The quaternionic analytic signal is defined by its partial (H_1, H_2) and total (H_T) Hilbert transforms (HT).

$$f_A(x, y) = f(x, y) + iH_1(f(x, y)) + jH_2(f(x, y)) + kH_T(f(x, y)) \quad (5)$$

where $H_1(f(x, y)) = f(x, y) ** \frac{\delta(y)}{\pi x}$, $H_2(f(x, y)) = f(x, y) ** \frac{\delta(x)}{\pi y}$ and $H_T(f(x, y)) = f(x, y) ** \frac{1}{\pi^2 xy}$. $\delta(y)$ and $\delta(x)$ are impulse sheets along x and y axis, respectively; and ** denotes 2-D convolution.

We start with real separable scaling function φ and mother wavelets ψ^H, ψ^V, ψ^D, for separable wavelet, $\psi(x, y) = \psi_h(x)\psi_h(y)$. According to the definition of the quaternionic analytic signal, the QWT, i.e. the analytic 2D wavelets can be constructed as follows.

$$\begin{cases} \varphi = \varphi_h(x)\varphi_h(y) \to \varphi + iH_1(\varphi) + jH_2(\varphi) + kH_T(\varphi) \\ \psi^H = \psi_h(x)\varphi_h(y) \to \psi^H + iH_1(\psi^H) + jH_2(\psi^H) + kH_T(\psi^H) \\ \psi^V = \varphi_h(x)\psi_h(y) \to \psi^V + iH_1(\psi^V) + jH_2(\psi^V) + kH_T(\psi^V) \\ \psi^D = \psi_h(x)\psi_h(y) \to \psi^D + iH_1(\psi^D) + jH_2(\psi^D) + kH_T(\psi^D) \end{cases} \quad (6)$$

For separable wavelet, $\psi(x, y) = \psi_h(x)\psi_h(y)$, 2-D HT equals to twice 1-D HT along row and column, respectively. Considering that 1-D HT pair, i.e. $(\psi_h, \psi_g = H\psi_h)$, and scaling function $(\phi_h, \phi_g = H\phi_h)$, 2-D analytic wavelet can be derived from formula (6) as the product form of 1-D separate wavelet,

$$\begin{cases} \varphi_A = \varphi_h(x)\varphi_h(y) + i\varphi_g(x)\varphi_h(y) + j\varphi_h(x)\varphi_g(y) + k\varphi_g(x)\varphi_g(y) \\ \psi_A^H = \psi_h(x)\varphi_h(y) + i\psi_g(x)\varphi_h(y) + j\psi_h(x)\varphi_g(y) + k\psi_g(x)\varphi_g(y) \\ \psi_A^V = \varphi_h(x)\psi_h(y) + i\varphi_g(x)\psi_h(y) + j\varphi_h(x)\psi_g(y) + k\varphi_g(x)\psi_g(y) \\ \psi_A^D = \psi_h(x)\psi_h(y) + i\psi_g(x)\psi_h(y) + j\psi_h(x)\psi_g(y) + k\psi_g(x)\psi_g(y) \end{cases} \quad (7)$$

The real-imaginary quaternion analytic form in (7) can be transformed into magnitude-phase form as (3) according to computational rules in (4) for the quaternion algebra. The QWT magnitude $|q|$, with the property of near shift-invariance, represents features at any spatial position in each frequency sub-band, and the three phases (ϕ, ψ, θ) describe the 'structure' of those features. In our paper, (ϕ, ψ, θ) of QWT is exploited to detect image smooth regions. More details about implementation of QW used here are referenced to the work [16].

3. Noise level estimation

3.1. Noise model and wavelet based noise level estimation

Let the image be $\mathbf{X}$, and the observed noisy image corrupted by additive noise is

$$\mathbf{Y} = \mathbf{X} + \mathbf{E} \quad (8)$$

where noise $\mathbf{E}$ is independent of $\mathbf{X}$ and satisfy the normal distribution $\mathbf{E} \sim \mathbf{N}(0, \sigma_n^2)$. Actually, the noise standard variance σ_n represents the noise level in our paper.

Most of the denoising methods necessitate estimating the noise variance as accurately as possible. Donoho proposed a well-known estimation method [13],

$$\sigma_n = \frac{Median\{|y(i,j)|\}}{0.6745}, \ y(i,j) \in HH_1 \quad (9)$$

which is a robust median estimation. After one level wavelet decomposing, coefficients belonging to the diagonal high frequency subband HH_1 are mainly consisted of noises. To avoid the interference of less useful signals with high value, the median operation is executed. The work [18] extended it into quaternion wavelet domain and proposed the corresponding Bayesian thresholding based denoising method which shows very good performance.

$$T = \gamma \frac{\sigma_n}{\sigma} \quad (10)$$