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Exact EM field excited by a short horizontal wire antenna lying on a conducting soil

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ABSTRACT

To date, only under restrictive assumptions can we derive closed-form expressions for the fields generated by an electrically small horizontal wire antenna situated on a planar conducting soil. Such assumptions limit the validity of the derived formulas to specified frequency ranges, or to the case of highly-conducting material media. The purpose of this work is to relax all the constraints underlying the derivation of the previously published solutions to this half-space problem, and develop an analytical technique that allows to reduce the integral representations for the field components generated by the dipole source to a well-known elementary contour integral, whose evaluation is straightforward. Numerical results are presented to show the advantages of the obtained explicit formulas, which are valid regardless of the operating frequency, over the previous solutions.

1. Introduction

Radiation from electrically small straight wire antennas placed above a flat, finitely conducting, homogeneous soil has been intensively investigated by many researchers beginning with Sommerfeld [1–16]. This is because the study has many practical applications, especially in the areas of radio communication and remote sensing [2–4,6,7,12–16]. Even though the work by Sommerfeld led to exact frequency-domain integral representations for the EM field components in and above the lossy medium, usage of the obtained expressions has revealed to be impractical, as numerical integration is made difficult by the highly oscillatory nature of the integrands. This is the reason why most scientists addressed the problem of performing analytical integration, after introducing restrictive assumptions aimed at making the field integrals tractable. Examples of approximate analytical solutions are the expressions proposed by Moore and Blair [2], Baños [3], Wait [5], and Bannister [6], which have been derived under the assumption that the ratio between the wavenumbers in free-space (k_0) and in the lossy medium (k_1) is far less than unity. The main drawback of these contributions is that each of them is valid only in a prescribed portion of the space surrounding the antenna (that is the near-, intermediate-, or far-field region).

In the nineties, King [7–10] was the first to derive a unique set of expressions, valid in a wide frequency range, for the time-harmonic fields of an electric type source in the presence of a plane interface between air and ground. The only disadvantage of King's formulation resides in that it is still subject to a condition on the ratio between the wavenumbers in free-space and in the conducting medium (namely $k_0^2/|k_1^2| \ll 1$), which cannot be met if the conductivity of the medium is low [5,17,18].

The present paper describes an analytical procedure that permits to derive rigorous expressions for the radial distributions of the EM field components of a horizontal electric dipole (HED) placed on a homogeneous soil. The procedure simply consists of casting the integral representations for the fields into forms involving only a well-known elementary contour integral, once the non-oscillating parts of the integrands are replaced with fast-convergent sequences of rational functions. Since the procedure does not require satisfaction of any condition, the obtained series representations for the fields are valid without restrictions on the operating frequency as well as the electromagnetic parameters of the material half-space.

The results originating from using the new analytical expressions agree well with the data generated by finite difference time domain (FDTD) simulations [19]. Conversely, disagreement with the outcomes provided by King's approach is observed when the skin depth in the medium is not negligible with respect to the source-receiver distance, that is at low frequencies, or when entering the far-field zone.

2. Theory

Consider a y-directed HED of moment $pe^{j\omega t}$ lying on the surface of a flat, homogeneous, isotropic and linear lossy medium. The EM

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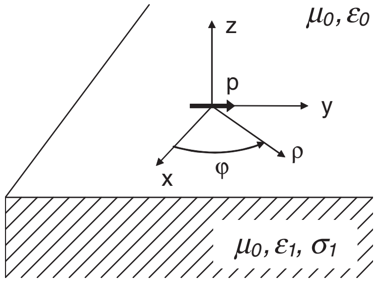

Fig. 1. Sketch of a horizontal electric dipole on a homogeneous lossy medium.

parameters of the medium are as depicted in Fig. 1, and a cylindrical coordinate system (ρ, φ, z) is introduced. The EM field components produced by the dipole at the air side of the air-medium interface may be expressed in compact form as [1]

$$E_\rho = \sin\varphi\left(k_0^2 + \frac{\partial^2}{\partial\rho^2}\right)\Pi_{y0} + \frac{\partial}{\partial\rho}\left.\frac{\partial\Pi_z}{\partial z}\right|_{z=0}, \tag{1}$$

$$E_\varphi = \cos\varphi\left(k_0^2 + \frac{1}{\rho}\frac{\partial}{\partial\rho}\right)\Pi_{y0} + \frac{\cot\varphi}{\rho}\left.\frac{\partial\Pi_z}{\partial z}\right|_{z=0}, \tag{2}$$

$$E_z = k_0^2\Pi_{z0} + \left.\frac{\partial^2\Pi_z}{\partial z^2}\right|_{z=0} + \sin\varphi\frac{\partial}{\partial\rho}\left.\frac{\partial\Pi_y}{\partial z}\right|_{z=0}, \tag{3}$$

$$H_\rho = j\omega\epsilon_0\left(-\cos\varphi\left.\frac{\partial\Pi_y}{\partial z}\right|_{z=0} + \frac{\cot\varphi}{\rho}\Pi_{z0}\right), \tag{4}$$

$$H_\varphi = j\omega\epsilon_0\left(\sin\varphi\left.\frac{\partial\Pi_y}{\partial z}\right|_{z=0} - \frac{\partial\Pi_{z0}}{\partial\rho}\right), \tag{5}$$

$$H_z = j\omega\epsilon_0\cos\varphi\frac{\partial\Pi_{y0}}{\partial\rho}, \tag{6}$$

where the subscript "0" denotes calculation at $z = 0^+$, and

$$\Pi_y = -\frac{p}{2\pi\omega\epsilon_0}\int_0^\infty \frac{e^{-jk_{0z}z}}{k_{0z}+k_{1z}}J_0(\lambda\rho)\lambda d\lambda, \tag{7}$$

$$\Pi_z = \frac{p\sin\varphi}{2\pi j\omega\epsilon_0 k_1^2}\frac{\partial}{\partial\rho}\int_0^\infty \frac{k_{0z}-k_{1z}}{k_{0z}+\tau^2 k_{1z}}e^{-jk_{0z}z}J_0(\lambda\rho)\lambda d\lambda \tag{8}$$

are the y- and z-components of the electric Hertz vector generated in the air-space, being $J_0(\cdot)$ the zeroth-order Bessel function, and

$$k_{nz} = \sqrt{k_n^2 - \lambda^2}, \quad Im[k_{nz}] < 0, \tag{9}$$

$$k_n^2 = \omega^2\mu_0\epsilon_n - j\omega\mu_0\sigma_n, \quad \tau = k_0/k_1. \tag{10}$$

The scope of the present paper is to exactly evaluate the EM field components (1)–(6). First, substituting the identities

$$\frac{\tau^2(k_{0z}-k_{1z})}{k_{0z}+\tau^2 k_{1z}} = \frac{\left(1+\tau^2\right)k_{0z}}{k_{0z}+\tau^2 k_{1z}} - 1, \tag{11}$$

$$\frac{k_{0z}(k_{0z}-k_{1z})}{k_{0z}+\tau^2 k_{1z}} = \frac{k_0^2}{k_{0z}+\tau^2 k_{1z}} - \frac{k_1^2}{k_{0z}+k_{1z}}, \tag{12}$$

respectively into (8) and its z-derivative, and taking into account that [1,14]

$$\int_0^\infty \frac{e^{-jk_{0z}z}}{k_{0z}}J_0(\lambda\rho)\lambda d\lambda = \frac{je^{-jk_0\sqrt{\rho^2+z^2}}}{\sqrt{\rho^2+z^2}}, \tag{13}$$

provides

$$\Pi_z = \frac{p\sin\varphi}{2\pi\omega\epsilon_0 k_0^2}\frac{\partial^2}{\partial\rho\partial z}\left[\int_0^\infty \frac{\left(1+\tau^2\right)e^{-jk_{0z}z}}{k_{0z}+\tau^2 k_{1z}}J_0(\lambda\rho)\lambda d\lambda - \frac{je^{-jk_0\sqrt{\rho^2+z^2}}}{\sqrt{\rho^2+z^2}}\right], \tag{14}$$

$$\frac{\partial\Pi_z}{\partial z} = -\sin\varphi\frac{\partial}{\partial\rho}\left[\frac{p}{2\pi\omega\epsilon_0}\int_0^\infty \frac{\tau^2 e^{-jk_{0z}z}}{k_{0z}+\tau^2 k_{1z}}J_0(\lambda\rho)\lambda d\lambda + \Pi_y\right]. \tag{15}$$

Next, using (7), (14) and (15) into (1)–(6) makes it possible to obtain

$$E_\rho = -\frac{\omega\mu_0 p\sin\varphi}{2\pi}\left(\frac{1}{k_1^2}U_0 + W_0\right),$$

$$H_\rho = \frac{jp\cos\varphi}{2\pi}\left(\frac{1}{\lambda_{01}^2\rho}V_0' + W_0'\right),$$

$$E_\varphi = -\frac{\omega\mu_0 p\cos\varphi}{2\pi}\left(\frac{1}{k_1^2\rho}V_0 + W_0\right),$$

$$H_\varphi = -\frac{jp\sin\varphi}{2\pi}\left(\frac{1}{\lambda_{01}^2}U_0' + W_0'\right),$$

$$E_z = \frac{\omega\mu_0 p\sin\varphi}{2\pi k_0^2}V_0', \quad H_z = -\frac{jp\cos\varphi}{2\pi}\frac{\partial W_0}{\partial\rho}, \tag{16}$$

with

$$U = \int_0^\infty \frac{e^{-jk_{0z}z}}{k_{0z}+\tau^2 k_{1z}}\left[-\lambda^2 J_0(\lambda\rho) + \frac{\lambda}{\rho}J_1(\lambda\rho)\right]\lambda d\lambda, \tag{17}$$

$$V = -\int_0^\infty \frac{e^{-jk_{0z}z}}{k_{0z}+\tau^2 k_{1z}}J_1(\lambda\rho)\lambda^2 d\lambda, \tag{18}$$

$$W = \int_0^\infty \frac{e^{-jk_{0z}z}}{k_{0z}+k_{1z}}J_0(\lambda\rho)\lambda d\lambda, \tag{19}$$

and

$$\lambda_{01} = -\frac{k_0}{\sqrt{1+\tau^2}}, \tag{20}$$

and where the subscript "0" and the prime denote, respectively, calculation at $z = 0^+$ and differentiation with respect to z. Explicit expressions for W_0 and W_0' are tabulated in literature. Applying [5, No. 73] leads, straightforwardly, to

$$W_0 = \frac{j(Q_1 - Q_0)}{\lambda_{00}^2}, \tag{21}$$

with

$$Q_n = (1 + jk_n\rho)\frac{e^{-jk_n\rho}}{\rho^3}, \quad \lambda_{00} = -k_1\sqrt{1-\tau^2}, \tag{22}$$

while from [20] it follows that

$$W_0' = \frac{j\lambda_{00}^2}{8}\left[K_2(\gamma\rho)I_2\left(\beta\rho\right) - K_0(\gamma\rho)I_0\left(\beta\rho\right)\right], \tag{23}$$

being $I_0(\cdot)$ and $K_0(\cdot)$ the zeroth-order modified Bessel functions of the first and second kind, respectively, and

$$\gamma = \frac{jk_1(1+\tau)}{2}, \quad \beta = \frac{jk_1(1-\tau)}{2}. \tag{24}$$

On the other hand, the integrals U_0, V_0, U_0' and V_0' may be calculated through a rigorous analytical approach. Use of [21, 3.2.62–3.2.68] allows to express (17), in the limit as $z \to 0$, as

$$U_0 = \frac{1}{2}\int_\Gamma \frac{1}{k_{0z}+\tau^2 k_{1z}}\left[-\lambda^2 H_0^{(1)}(\lambda\rho) + \frac{\lambda}{\rho}H_1^{(1)}(\lambda\rho)\right]\lambda d\lambda, \tag{25}$$