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A variational model based on split Bregman method for multiplicative noise removal☆

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ABSTRACT

It is well known that total variation (TV) regularizer leads to the staircase effect, the higher order variational methods give rise to the restored image blurred. In this paper, we propose a novel variational model for multiplicative noise removal. The proposed model can automatically adjust the first and second order regularization terms. To solve such an objective function effectively, the split Bregman and primal-dual methods are employed in our numerical algorithm. Our experimental results show that the proposed method is more effective to filter out the multiplicative noise compared with the recent methods.

1. Introduction

Image denoising is one of the fundamental problems in image processing and computer vision fields. For many special imaging systems such as synthetic aperture radar, laser or ultrasound imaging, and positron emission tomography, image acquisition processes are different from the usual optical imaging technology, and they may be distorted by some unexpected random noise and the standard additive Gaussian noise model is not suited in these situations. However, the multiplicative noise model provides an appropriate description of these special imaging systems. In this paper, we focus on the issue of multiplicative noise removal. The noise model is

$$f_0 = f\eta, \tag{1}$$

where f_0 is the observed image, f is the original image, η is the noise which follows a Gamma Law with mean one and its probability density function is given by

$$g(\eta) = \frac{L^L}{\Gamma(L)}\eta^{L-1}\exp(-L\eta), \tag{2}$$

where L is the number of looks (in general, an integer coefficient), and $\Gamma(\cdot)$ is a Gamma function. Multiplicative noise is one of the complex noise models. It is signal independent, non-Gaussian, and spatially dependent. Hence, multiplicative noise removal is a very challenging problem compared with additive Gaussian noise.

For tackling the problem, many approaches have been proposed. Popular methods include the Lee method [1], the multiscale shrinkage methods [2], various anisotropic diffusion based methods [3,4], and variational methods [5–15]. The first TV based multiplicative noise removal model was presented by Rudin et al. [7], which used a constrained optimization approach with two Lagrange multipliers. Following the maximum a posteriori estimator for multiplicative Gamma noise, Aubert and Aujol [8] introduced a non-convex model (AA)

$$\min_f \left\{ \|f\|_{TV} + \lambda \int_\Omega \left(\log f + \frac{f_0}{f} \right) dx \right\}, \tag{3}$$

where $\|f\|_{TV} = \int_\Omega |\nabla f|\, dx$ is the TV regularization term, $\int_\Omega (\log f + (f_0/f))\, dx$ is the data fidelity term, and $\lambda > 0$ is the regularization parameter. However, the data fidelity term is non-convex, it dose not have the global minimal solution. By the noisy observation, Shi and Osher [9] derived a strictly convex model (SO)

$$\min_\omega \left\{ \|\omega\|_{TV} + \lambda \int_\Omega (f_0 \exp(-\omega) + \omega)\, dx \right\}, \tag{4}$$

where $\omega = \log f$. To solve the model, the authors applied a relaxed inverse scale space flow method. Although this method has an excellent denoising effect and significant improvement over earlier multiplicative noise removal models, it needs a lot of time to run the iteration. To improve the speed of operation, a new variational model was introduced by Huang et al. [10] through

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variable splitting. The corresponding minimization problem was given by

$$\min_{z,\omega}\|\omega\|_{TV}+\lambda\int_{\Omega}(f_0\exp(-z)+z)\,dx+\mu\|z-\omega\|_2^2. \quad (5)$$

Eq. (5) was solved by using an alternating minimization method and the estimated restoration image $f^*=\exp(z^*)$. The proposed method is fast, since the subproblem with respect to z is fast solved by using Newton's method; the subproblem with respect to ω is typical ROF denoising problem, which is efficiently solved by dual method. However, Eq. (5) becomes severely ill-conditioned when μ is very large. Considering such a difficulty, Bioucas and Figueiredo [11] proposed a new multiplicative noise removal model by combing operator splitting with augmented Lagrangian method. In addition, Steidl and Teuber [12] also proposed a convex model, in which the I-divergence is used as the fidelity term. In order to use the local statistical characteristics of image, Chen et al. [13] proposed a spatially adapted total variational model to remove multiplicative noise. The advantage of this model is that the regularization parameters are automatically selected and the image details were preserved well.

Recently, sparse variational models have been widely applied to image processing because they can preserve the geometrical structures of the restoration images very well. Durand,et al. [14] proposed a hybrid method composed of an l_1 data-fitting for the curvelet frame coefficients and TV. Combined the weighted TV with the data terms in Eq. (4), a non-convex sparse regularizer variational model is proposed [15]. The advantage of this model is that it overcomes the disadvantage of the non-convexity and turns the non-convex variational model into several convex ones by use of the augmented Lagrange multiplier method and the iteratively reweighted method.

However, the TV regularization suffers from the so-called staircase effect, namely, the transformation of smooth regions into piecewise constant ones, which may produce undesirable blocky images. In order to reduce the staircase effect, some higher order partial differential equations (PDEs) were introduced [16,17]. These methods are based on minimizing the total variation of image gradient or the derivatives of the image rather than the total variation of the image itself. However, the fourth-order PDEs can make the edges and textures of those restoration images blurry.

In recent years, some lower order variational models have started to appear as the very efficient methods for image denoising [18–25], which are capable of restoring edges and discontinuities in a better way than the fourth-order PDE. Inspired by the idea in [25], we will give a novel variational model for multiplicative noise removal in this paper. The new model takes advantage of the restored log image information to remove noise. In addition, to solve the proposed model effectively, we design the split Bregman and primal-dual algorithms. From the experimental results, we see that the new model is more effective for multiplicative noise removal.

The outline of this paper is as follows. In Section 2, a new variational model and an effective algorithm are designed for multiplicative noise removal. In Section 3, some experimental results are demonstrated the qualities of the restored images. Finally, conclusions are given in Section 4.

2. The proposed model and algorithm for multiplicative noise removal

2.1. The proposed model

In [25], the authors proposed a variational model using the L_1 fidelity term of the gradient and a vector field and successfully alleviated the staircase effect for the additive Gaussian noise. So inspired by this idea, we let $\boldsymbol{v}$ be the vector field of the log image, and propose the following variational model for multiplicative noise removal:

$$\min_{\omega,v}\alpha_1\int_{\Omega}\left|\nabla\omega-\mathbf{v}\right|dx+\alpha_2\int_{\Omega}\left|\nabla\boldsymbol{v}\right|dx+\int_{\Omega}(f_0\exp(-\omega)+\omega)\,dx, \quad (6)$$

where $\omega=\log f$, the first term and the third term is the fidelity term, the second term is the regularization term. $\alpha 1, \alpha_2$ are non-negative regularization parameters.

Remark some properties of the proposed model. First, the proposed model has only one energy functional which is different from [21–23], ω and $\boldsymbol{v}$ can be mutually made full use of each other, that is, the smoothed vector field $\boldsymbol{v}$ depends on the restored image ω. In addition, $\boldsymbol{v}$ is a vector field of the log image, not the vector field of the original image, which is different from [25].

Second, from the first and second terms of the proposed model, we can intuitively see that when $\boldsymbol{v}$ approaches $\nabla\omega$ and the above two terms turn into the second order derivative term; when $\boldsymbol{v}$ approaches zero, the above two terms turn into the total variational regularization term. So compared with Eqs. (3)–(5) and the model in [15], the proposed model can automatically balance the first and second order derivatives through parameters α_1 and α_2. This balancing leads to edges penalty not being larger than a smooth function, which is similar to TV, but to smooth regions being less than the staircase effect which is absent in TV.

From the above explanations, we can conclude that the proposed model provides a way of balancing between the first and second order derivatives of a function, so it can reduce the staircase effect while denoising, meanwhile, it has the property of edge preservation which is very important in image processing.

2.2. The proposed algorithm

Recently, the split Bregman method introduced by Goldstein and Osher [26] has become a very effective tool for solving various problems including L_1 norm. The method has been proven to be equivalent to the augmented Lagrangian method [27,28] and also belongs to the framework of the Douglas-Rachford splitting algorithm [29]. With the advantages such as fast convergence speed, numerical stabilities and smaller memory footprint, etc., see details in Goldstein and Osher [26], the split Bregman method has been used widely in image processing. In the following, we shall use it to solve Eq. (6).

Firstly, we introduce a variable z in Eq. (6), and let $z=\omega$. Then we can get

$$\min_{\omega,v,z}\alpha_1\int_{\Omega}\left|\nabla z-\mathbf{v}\right|dx+\alpha_2\int_{\Omega}\left|\nabla\boldsymbol{v}\right|dx + \int_{\Omega}(f_0\exp(-\omega)+\omega)\,dx,\quad s.t.\quad z=\omega. \quad (7)$$

Introducing the auxiliary variable b and using the split Bregman method, we have the following subproblems to solve:

$$(z^{k+1},\boldsymbol{v}^{k+1})=\operatorname{argmin}_{z,v}\alpha_1\int_{\Omega}\left|\nabla z-v\right|dx +\alpha_2\int_{\Omega}\left|\nabla\boldsymbol{v}\right|dx+\frac{\beta}{2}\left\|z-\omega^k-b^k\right\|_2^2, \quad (8)$$

$$\omega^{k+1}=\operatorname{argmin}_{\omega}\frac{\beta}{2}\left\|\omega+b^k-z^{k+1}\right\|_2^2+\int_{\Omega}(f_0\exp(-\omega)+\omega)\,dx, \quad (9)$$

$$b^{k+1}=b^k-z^{k+1}+\omega^{k+1}. \quad (10)$$