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Scattering from a chirally coated DB elliptic cylinder

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ABSTRACT

An exact analytic solution to the problem of scattering of a plane electromagnetic wave from a chirally coated elliptic cylinder defined by a DB boundary has been obtained, by expanding the different electromagnetic fields associated with the problem in terms of suitable elliptic vector wave functions and a set of expansion coefficients. The incident field expansion coefficients are known, but the expansion coefficients associated with the fields scattered outside the coated cylinder and the fields transmitted inside the coating are unknown. These unknown coefficients are obtained by imposing appropriate boundary conditions at the two boundaries. Results have been presented as normalized bistatic and backscattering widths for a variety of admittances, permeabilities, and permittivities of the chiral materials used for the coating, to show their effects on scattering from the chirally coated cylinder.

1. Introduction

Over the years, there had been a lot of research conducted on scattering from elliptic cylinders coated with different materials. Such interest has mainly been due to the ability to model or approximate coated cylindrical objects of different cross sectional shapes using coated elliptic cylinders of suitable axial ratios to investigate how scattering from them can be controlled using the material parameters and thickness of the coating, and to obtain an exact analytic solution to the scattering problem using the method of separation of variables. Scattering from elliptic cylinders with a single layer coating [1–9] as well as from those with a multi-layer coating [10–12] have been analyzed in the literature.

An alternative set of boundary conditions applicable to electromagnetic problems known as DB boundary conditions which require vanishing of the normal components of electric and magnetic flux densities on the boundary [13], has attracted a lot of interest lately due to their use in the solution of problems associated with invisibility cloaks [14–17]. A detailed study of these boundary conditions has been conducted in [18–21]. A circular waveguide defined by a DB boundary has been analyzed in [22], and plane-wave reflections from a planar DB boundary have been considered in [23,24]. Scattering of a normally incident plane wave from a circular cylinder defined by a DB boundary and uniformly coated with a chiral material has recently been presented in [25]. Here, we consider for the first time, the more general case of scattering of an arbitrarily incident plane wave from an elliptic cylinder defined by a DB boundary (hereafter called a DB elliptic cylinder) and confocally coated with a chiral material. The primary motivation for this new research is the capability to obtain an exact analytic solution to the problem, to be used as a benchmark. One secondary motivation is the capacity to model or approximate coated cylindrical objects of various cross sectional shapes more accurately using coated elliptic cylinders of variable axial ratios instead of coated circular cylinders, and to exploit the chirality of the coating material as an extra degree of freedom to control scattering from the chirally coated cylinder in addition to the constitutive parameters and thickness of the coating. The other secondary motivation is the potential to use the given formulation in the designing of invisibility cloaks.

Since the elliptic coordinate system is one of the coordinate systems under which the wave equation is separable, the problem has been formulated by expressing the fields associated with the problem in terms of suitable elliptic vector wave functions, using the method of separation of variables. Also, the analysis and the software used for obtaining results have been validated, by computing normalized bistatic and backscattering widths of a chirally coated DB elliptic cylinder of axial ratio 1.0001, comparing these with the same obtained for a corresponding chirally coated DB circular cylinder by analyzing it using circular cylindrical vector wave functions, and showing that they are in excellent agreement.

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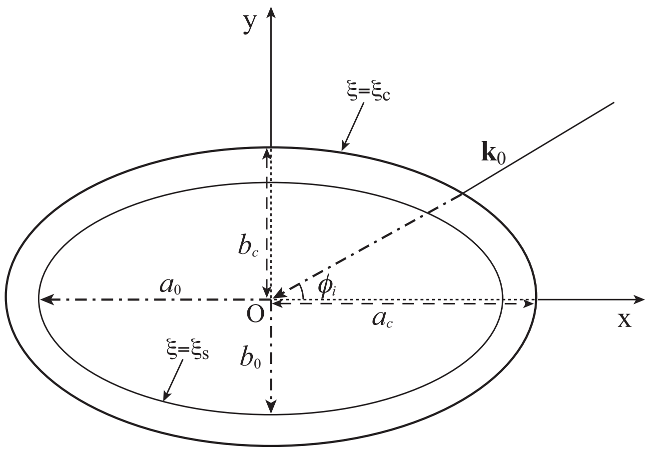

Fig. 1. Geometry of the coated DB elliptic cylinder.

2. Formulation

Consider a linearly polarized uniform plane electromagnetic wave arbitrarily incident on an infinitely long DB elliptic cylinder confocally coated with a chiral material. The semi-major and semi-minor axis lengths of the uncoated cylinder are denoted by a_0 and b_0, and those of the coated cylinder are denoted by a_c and b_c, respectively. The coated cylinder is assumed to be located in free space, with the incident wave making an angle ϕ_i with the negative x-axis of a Cartesian coordinate system located at the centre of the elliptic face as shown in Fig. 1, and the axis of the cylinder along the negative z-axis. From the point of view of the analysis, it is beneficial to define the x and y coordinates of the Cartesian coordinate system in terms of the u, v, z coordinates of an elliptical coordinate system also located at the centre of the elliptic face as $x = F\cosh u \cos v$, $y = F\sinh u \sin v$, with F being the semi-focal length of the ellipse. A time harmonic dependence of $\exp(j\omega t)$ with ω being the angular frequency is assumed throughout the analysis, but suppressed for convenience. The analysis is conducted for an incident plane wave of transverse magnetic (TM) polarization. The analysis corresponding to an incident plane wave of transverse electric (TE) polarization can be obtained from that for the plane wave of TM polarization, using duality.

2.1. Incident field

For a transverse magnetically polarized incident plane wave of unit electric field amplitude, the incident electric and magnetic fields can be expanded as [26]

$$\mathbf{E}_i = \sum_{q=e,o}\sum_{n=0}^{\infty} A_{qn}\mathbf{N}_{qn}^{(1)}(c_0, \boldsymbol{r}) = \sum_{q,n} A_{qn}\mathbf{N}_{qn}^{(1)}(c_0, \boldsymbol{r}) \quad (1)$$

$$\mathbf{H}_i = \frac{j}{Z_0}\sum_{q=e,o}\sum_{n=0}^{\infty} A_{qn}\mathbf{M}_{qn}^{(1)}(c_0, \boldsymbol{r}) = \frac{j}{Z_0}\sum_{q,n} A_{qn}\mathbf{M}_{qn}^{(1)}(c_0, \boldsymbol{r}) \quad (2)$$

where Z_0 is the free space wave impedance, $c_0 = k_0F$, with k_0 being the free space wavenumber, and $\boldsymbol{r}$ denotes the coordinate dyad ξ, η with $\xi = \cosh u$, $\eta = \cos v$. The even (e) and odd (o) elliptic vector wave functions $\mathbf{M}$ and $\mathbf{N}$ in (1) and (2) are given by

$$\mathbf{N}_{qn}^{(i)}(c, \boldsymbol{r}) = \hat{\boldsymbol{z}} R_{qn}^{(i)}(c, \xi) S_{qn}(c, \eta) \quad (3)$$

$$\mathbf{M}_{qn}^{(i)}(c, \boldsymbol{r}) = \hat{\boldsymbol{u}} \frac{R_{qn}^{(i)}(c, \xi) S'_{qn}(c, \eta)}{kh} - \hat{v} \frac{R_{qn}^{(i)\prime}(c, \xi) S_{qn}(c, \eta)}{kh} \quad (4)$$

in which S_{qn} and $R_{qn}^{(i)}$ for $q = e, o$ are the even and odd angular Mathieu functions and radial Mathieu functions of the ith kind, both of order n, respectively, the primes on S and R denoting their first derivatives with respect to v and u, respectively, $c = kF$ with k being the wavenumber, $\hat{\boldsymbol{\tau}}$ denotes a unit vector in the positive τ-direction, and $h = F\sqrt{\xi^2 - \eta^2}$. In (1) and (2), the double summation over q and n has been expressed as a single summation over q, n for convenience in writing. This notation will be used throughout the rest of the paper.

The expansion coefficients in (1) and (2) are given by

$$A_{qn} = j^n \frac{\sqrt{8\pi}}{N_{qn}(c_0)} S_{qn}(c_0, \cos\phi_i) \quad (5)$$

with $N_{qn}(c)$ being the normalization constant of order n associated with $S_{qn}(c, \eta)$ [27]. The proposed method of analysis can handle arbitrarily incident waves, since the incident electric and magnetic fields expanded in (1) and (2) using the incident field expansion coefficients A_{qn} in (5), are a function of the incident angle ϕ_i.

2.2. Scattered field

The electric and magnetic fields scattered by the chirally coated DB cylinder in response to the incident plane wave, consist of co-polar as well as cross polar components. Thus, they can be expanded as a superposition of TE and TM waves in the form [25]

$$\mathbf{E}_s = \sum_{q,n}[B_{qn}\mathbf{N}_{qn}^{(4)}(c_0, \boldsymbol{r}) + C_{qn}\mathbf{M}_{qn}^{(4)}(c_0, \boldsymbol{r})] \quad (6)$$

$$\mathbf{H}_s = \frac{j}{Z_0}\sum_{q,n}[B_{qn}\mathbf{M}_{qn}^{(4)}(c_0, \boldsymbol{r}) + C_{qn}\mathbf{N}_{qn}^{(4)}(c_0, \boldsymbol{r})] \quad (7)$$

in which B_{qn} and C_{qn} are the unknown scattered field expansion coefficients.

2.3. Transmitted field

Since the electric and magnetic fields transmitted inside the chiral coating consist of left- and right-handed circularly polarized waves, they can be expanded as [28]

$$\mathbf{E}_c = \sum_{q,n}[P_{qn}^{(1)}\boldsymbol{\Xi}_{qn}^{+(1)}(c_R, \boldsymbol{r}) + Q_{qn}^{(1)}\boldsymbol{\Xi}_{qn}^{-(1)}(c_L, \boldsymbol{r}) + P_{qn}^{(2)}\boldsymbol{\Xi}_{qn}^{+(2)}(c_R, \boldsymbol{r}) + Q_{qn}^{(2)}\boldsymbol{\Xi}_{qn}^{-(2)}(c_L, \boldsymbol{r})] \quad (8)$$

$$\mathbf{H}_c = \frac{j}{Z_c}\sum_{q,n}[P_{qn}^{(1)}\boldsymbol{\Xi}_{qn}^{+(1)}(c_R, \boldsymbol{r}) - Q_{qn}^{(1)}\boldsymbol{\Xi}_{qn}^{-(1)}(c_L, \boldsymbol{r}) + P_{qn}^{(2)}\boldsymbol{\Xi}_{qn}^{+(2)}(c_R, \boldsymbol{r}) - Q_{qn}^{(2)}\boldsymbol{\Xi}_{qn}^{-(2)}(c_L, \boldsymbol{r})] \quad (9)$$

where $P_{qn}^{(i)}$, $Q_{qn}^{(i)}$ for $i = 1, 2$ are the unknown field expansion coefficients, $\boldsymbol{\Xi}_{qn}^{\pm(i)}(c_\alpha, \boldsymbol{r}) = \mathbf{N}_{qn}^{\pm(i)}(c_\alpha, \boldsymbol{r}) \pm \mathbf{M}_{qn}^{\pm(i)}(c_\alpha, \boldsymbol{r})$, $c_R = k_RF$, $c_L = k_LF$, with the wavenumbers k_R and k_L corresponding to the right- and left-handed waves inside the chiral medium given by $k_{R,L} = \omega\sqrt{\mu\epsilon_c} \pm \omega\mu\zeta_c$, in which ζ_c is the chirality admittance and ϵ_c is the effective permittivity defined by $\epsilon_c = \epsilon + \mu\zeta_c^2$ with ϵ and μ being the permittivity and the permeability of the chiral medium, and Z_c is the wave impedance in the chiral medium, given by $Z_c = \sqrt{\mu/\epsilon_c}$ [29].

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