Int. J. Electron. Commun. (AEÜ) 68 (2014) 1112–1118

Contents lists available at ScienceDirect

International Journal of Electronics and Communications (AEÜ)

journal homepage: www.elsevier.com/locate/aeue

Steady-state analysis of a complex adaptive notch filter using modified gradient algorithm

R. Punchalard *

Department of Telecommunication Engineering, Mahanakorn University of Technology, Bangkok 10530, Thailand

ARTICLE INFO

Article history:
Received 20 June 2013
Accepted 30 May 2014

Keywords:
Complex notch filter
Gradient algorithm
Frequency estimation
Power spectral density

ABSTRACT

Based on power spectral density (PSD) analytical technique, mean square error (MSE) (or variance) of the frequency estimate of a first-order complex adaptive IIR notch filter (ANF) using modified complex plain gradient (MCPG) algorithm is investigated in this paper. The steady-state expression for MSE is derived in closed form. A quantitative analysis for the estimation MSE has been carried out. It has been revealed that the MSE of frequency estimate is independent of an input frequency of a complex sinusoid. In addition, computer simulations are treated to corroborate the theoretical analysis and the relationships between MSE and system parameters are shown.

1. Introduction

Frequency estimation based on adaptive methods play a major role in many areas of digital signal processing applications [1,2] such as Doppler estimation of radar and sonar wave returns, carrier and clock synchronization, angle of arrival estimation, frequency-shift keying (FSK) signal demodulation and so on. There are plenty of methods that can be used to estimate the frequency of a complex sinusoid [3]. Complex ANF methods are widely used due to low complexity [4–12]. One example is to reject interference in the quadrature phase-shift keying (QPSK) spread spectrum by complex ANF [4,5]. The ANF that we have considered in this paper is the first-order complex ANF with constrained poles and zeros, which was first proposed by Pei [5]. This structure is widely used due to simplicity and economy. We can find its benefits in open literatures [6–9]. In addition, Regalia [11] has proposed the complex ANF which has the same system function as that of [5] but it is realized by using state-space approach instead of direct form realization.

Meanwhile, various adaptive algorithms have been developed for the constrained first-order complex ANF [5–12], such as the modified complex Gauss-Newton (MGN) algorithm [5], the complex plain gradient (CPG) algorithm [6–9], the normalized CPG (NCPG) algorithm [10], Regalia algorithm (RA) [11], and the recent adopted modified CPG (MCPG) algorithm [12]. The studies of those algorithms have revealed that the MGN is very complicated and it provides bias in the frequency estimate. Moreover, the stability check must be performed at all times. Next, the CPG has a main drawback which is that it has very slow adaptation speed when the optimum solution is at some distance from the initial value. This is because the slope of the cost function is nearly constant in regions away from the optimum. However, this algorithm provides unbiased frequency estimation. For the RA, although it provides fast adaptation speed as compared with the CPG, high fluctuation in the frequency estimate is also obtained. Moreover, it is difficult to select the appropriated pole radius ρ that can attain the highest performance of the algorithm. Finally, the MCPG provides not only fast adaptation, but also low computational complexity as compared with the comparative algorithms. Therefore, the MCPG can be considered as a candidate among other algorithms for real-time applications including complex sinusoidal frequency estimation and tracking in radar and sonar systems [5] and rejection of narrowband interference in QPSK spread spectrum [5] and QPSK systems [13]. In [12], however, only the derivation of the MCPG and mean value of the frequency estimate are mentioned but the steady-state MSE of the frequency estimate has not been considered yet. In [15], we have proposed the MSE analysis of the unbiased modified plain gradient (UMPG) algorithm for a real second-order ANF where the accurate analytical results were obtained. Unfortunately, we have found that it is difficult to apply the technique adopted in [15] to the case of complex ANF using the MCPG because of complex nature of the ANF and algorithm. Instead, we have found that the theoretical framework adopted in [10] is more simple and suitable to analyze the MCPG.

In this paper, performance assessment of the MCPG is extended to the MSE analysis. We apply the theoretical framework in [10] to the case of a complex ANF using the MCPG. The closed form

* Tel.: +66 2 9883655; fax: +66 2 9884040.
E-mail addresses: abubak_pk@yahoo.com, rachu@mut.ac.th

http://dx.doi.org/10.1016/j.aeue.2014.05.017
1434-8411/

expression for steady-state MSE of the frequency estimate is derived. Finally, computer simulations are conducted to corroborate the theoretical analysis and to demonstrate its comparative performances with some previous algorithms including the CPG and RA.

2. Complex ANF and MCPG

Let us define a noisy complex sinusoid $x(k)$ of amplitude A, frequency ω_0, and phase θ, of the form

$$x(k) = Ae^{j(\omega_0 k+\theta)} + n(k), \quad k = 0, 1, 2, \ldots \tag{1}$$

herein, $A > 0$ and $\omega_0 \in (0, \pi)$ are considered as deterministic but unknown constants. $\theta \in [0, 2\pi)$ is a uniform random variable. $n(k) = n_R(k) + jn_I(k)$ is assumed to be a zero-mean complex white process, where $n_R(k)$ and $n_I(k)$ are zero-mean real white processes with identical but unknown variances of $\sigma_n^2/2$ and uncorrelated with each other. The signal to noise ratio (SNR) can be calculated by

$$\text{SNR} = \frac{A^2}{\sigma_n^2}. \tag{2}$$

The MCPG [12] that is used to estimate an input frequency ω_0 of $x(k)$ is defined by

$$\Omega_0(k+1) = \Omega_0(k) + \mu \text{Re}\{e(k)g^*(k)\}, \tag{3}$$

where $\Omega_0(k)$ is the estimate of ω_0 at time instant k and μ is the stepsize parameter that is a positive real constant controlling the speed of convergence of the algorithm. Re$\{\cdot\}$ is the real part, $e(k)$ is known as the error or output signal of a complex ANF and is generated by the system function [12] of the form

$$H_e(z) \triangleq \frac{N(z)}{D(z)} = \frac{1 - e^{j\Omega_0}z^{-1}}{1 - \rho e^{j\Omega_0}z^{-1}}, \tag{4}$$

where ρ is the pole constraction factor determining the notch bandwidth, $N(z)$ and $1/D(z)$ are, respectively, all zeros and all poles systems and Ω_0 is the frequency parameter of the filter which is adjusted by (3). As discussed in [12], the MCPG is the simplified version of the CPG [6]. It provides better convergence property than that of the CPG. In addition, it has computational efficiency, is simple to implement and is appropriate for real-time applications. $g(k)$ is known as the gradient signal [12] and is given by

$$g(k) = je^{j\Omega_0}x(k-1). \tag{5}$$

It is easy to see that $g(k)$ is generated by the gradient filter of the form

$$H_g(z) = je^{j\Omega_0}z^{-1}. \tag{6}$$

In the next section, the steady-state analysis of the MCPG is addressed and derived in terms of the closed form expression for steady-state MSE of the frequency estimate $\Omega_0(k)$.

3. Steady-state analysis

In this section, performance analysis of the MCPG is addressed. Since we know that when the adaptive algorithm reaches its steady-state, the notch frequency $\Omega_0(k)$ coincides the frequency ω_0 of input sinusoid. However, due to input noise $n(k)$, the steady-state frequency variable will fluctuate around its true value, leading to mean square error of the estimated frequency. The MSE of the estimated parameter of any adaptive filter is an important quantity because it can be used to measure an accuracy of the estimated parameter. Thus prior to analyze the MSE of the frequency estimate, we first refer to the ensemble averaged value of the learning increment of (3), assuming that $\Omega_0(k)$ very slowly changes over time (see [12], (23)):

$$E[\text{Re}\{e(k)g^*(k)\}] = \text{Re}\{E[e(k)g^*(k)]\} = \frac{A^2(2\rho\cos(\Omega_0-\omega_0) - \rho - 1)\sin(\Omega_0-\omega_0)}{1+\rho^2-2\rho\cos(\Omega_0-\omega_0)}. \tag{7}$$

Substituting (7) into (3) yields the difference equation for the convergence in the mean of the frequency estimate $\Omega_0(k)$:

$$\overline{\Omega}_0(k+1) = \overline{\Omega}_0(k) + \mu\frac{A^2(2\rho\cos(\overline{\Omega}_0(k)-\omega_0) - \rho - 1)\sin(\overline{\Omega}_0(k)-\omega_0)}{1+\rho^2-2\rho\cos(\overline{\Omega}_0(k)-\omega_0)}, \tag{8}$$

where $\Omega_0(k)$ in (3) is replaced by $\overline{\Omega}_0(k) \triangleq E[\Omega_0(k)]$ for notation simplicity. From (7), it is obvious that $\Omega_0 = \omega_0$ is a stationary point. At steady-state where $\Omega_0 \simeq \omega_0$, $\cos(w)|_{w\to 0} \simeq 1$ and $\sin(w)|_{w\to 0} \simeq w$, (7) becomes

$$E[\text{Re}\{e(k)g^*(k)\}] \approx -\alpha A^2(\Omega_0 - \omega_0), \tag{9}$$

where $\alpha = 1/(1-\rho)$. Using (9) in (8), we obtain

$$\overline{\Omega}_0(k+1) = \overline{\Omega}_0(k) - \mu\alpha A^2(\overline{\Omega}_0(k) - \omega_0). \tag{10}$$

Eq. (10) is the first-order time-invariant difference equation in $\overline{\Omega}_0(k)$ whose solution is given by (see Appendix A)

$$\overline{\Omega}_0(k) = (\overline{\Omega}_0(-1) - \omega_0)(1-\mu\alpha A^2)^{k+1} + \omega_0, \tag{11}$$

where $\overline{\Omega}_0(-1)$ is an initial value of $\overline{\Omega}_0(k)$ at time instant $k = -1$. It is seen that the term $(1-\mu\alpha A^2)$ appeared on the right-hand side (RHS) of (11) tends to zero when $k \to \infty$ and (11) can then be rewritten as follows:

$$\overline{\Omega}_0(k)|_{k\to\infty} = \omega_0. \tag{12}$$

Therefore it has been proved from (12) that the MCPG is unbiased. Referring to (11), we can predict the iteration number N_i that is necessary to shift $\overline{\Omega}_0(k)$ from $\overline{\Omega}_0(-1)$ to ω_0 as follows. It is well-known that the term $(1-\mu\alpha A^2)$ exponentially decreases in time provided that ρ is sufficient small. Thus we can write that

$$1 - \mu\alpha A^2 = e^{-(1/\tau)}, \tag{13}$$

where τ is defined as the time constant and is derived from (13) as

$$\tau = -\frac{1}{\log_e(1-\mu\alpha A^2)}. \tag{14}$$

As a result, the approximated convergence time N_i of the MCPG therefore is

$$N_i \approx 5\tau\,(\text{samples}). \tag{15}$$

In addition, the stability bound of the stepsize in the sense of mean is easily derived from (11) as

$$0 < \mu < \frac{2}{\alpha A^2}. \tag{16}$$

The upper bound of stepsize in (16) guarantees monotonic convergence in the sense of mean [12].

Now, the steady-state MSE analysis of the frequency estimate $\Omega_0(k)$ based on the PSD technique [10] is introduced. Firstly, substituting $z = e^{j\omega_0}$, $\omega_0 \in [0, \pi]$ in (4) and (6) yields