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Harmonic and intermodulation performance of moderate inversion MOSFET transconductors

Muhammad Taher Abuelma'atti *

King Fahd University of Petroleum and Minerals, Box 203, Dhahran 31261, Saudi Arabia

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ABSTRACT

This article discusses the harmonic and intermodulation performance of moderate inversion MOSFET transconductors. The bulk of the nMOS transistor is tied to ground, at all levels of inversion, including moderate inversion and the transistor is operating in the saturation region where it behaves qualitatively as a constant current source. The current–voltage characteristic of the transistor is approximated using a Fourier-series model. Using this model, analytical expressions are obtained for amplitudes of the harmonics and intermodulation products resulting from multi-sinusoidal gate-to-source input voltages. The special case of a two equal-amplitude sinusoidal input is considered in detail and the results are compared with previously published results.

1. Introduction

MOSFET linear and squarer transconductors are widely used in the design of many analog blocks; see for example [1–5] and the references cited therein. Most of the MOSFET-based transconductors/squarer circuits are designed assuming the transistors are saturated and working in strong inversion; see for example [4–6]. Recently, however, it has been shown that operating the MOSFETs in moderate and weak inversion regions can minimize third-harmonic distortion; see for example [7–12]. Of particular interest here is the work reported in [11,12] that demonstrates the existence of possible sweet spots in the moderate inversion region. According to [11,12] these sweet spots can be exploited to minimize fourth harmonic distortion. Minimization of fourth harmonic distortion would result in a near ideal squaring function and help obtain high performance analog multipliers and true squarer circuits. In [11,12], it is assumed that the MOSFETs are modeled by the Enz–Krummanacher–Vittoz (EKV) model [13] shown in Eq. (1).

$$I_d = SI_s \left[\ln(1 + \exp\left(\frac{(V_{gs} - V_T)}{(2nV_t)} \right) \right]^2 \quad (1)$$

In Eq. (1), I_d is the drain current, $S \equiv W/L$ is the transistor strength ratio, $I_s \equiv 2\mu n C_{ox} V_t^2$ is the transistor specific current, μ is the electron mobility, $n = (C_{ox} + C_{dep})/C_{ox}$ is the subthreshold slope factor, V_t is the thermal voltage, V_{gs} is the gate-to-source voltage and V_T is the threshold voltage. Eq. (1) provides a model for the current–voltage characteristics of the nMOS transistor, whose bulk is tied to ground, at all levels of inversion, including moderate inversion. Eq. (1) assumes that the transistor is operating in the saturation region where it behaves qualitatively as a constant current source. While Eq. (1) has a limited accuracy, it can be used for providing a useful analytical tool for the analysis of harmonic and intermodulation in moderate inversion MOSFET transconductors. However, in its present form Eq. (1) cannot provide analytical expressions for the relative harmonic and intermodulation products. Recourse to Taylor series expansion was, therefore, inevitable in order to find the zeros of the third- and fourth-order derivatives of Eq. (1) in order to find the sweet points, where the harmonic distortion is minimized [11,12]. However, by virtue of its derivation, the results obtained in [11,12] are valid only under small signal conditions and cannot predict the harmonic and intermodulation performance of the transistor under large signal conditions. It is, therefore, the purpose of this paper to present an approximate model for Eq. (1). The model is valid over a wide range of input voltage and can, therefore, be used to predict the sweet points even under large signal conditions.

2. Proposed model

Here we propose to represent Eq. (1) by using the Fourier-series model of Eq. (2).

$$y = \Delta + \sum_{n=1}^{N} b_n \sin\left(\frac{2n\pi}{D} x \right) \quad (2)$$

In Eq. (2), $y = I_d/SI_s$ represents the normalized drain current, $x = (V_{gs} - V_T)/(2nV_t)$ represents the normalized gate-to-source voltage, Δ represents the offset at $x = 0$. The parameters D and b_n, $n = 1$,

* Tel.: +966 3 860 2796; fax: +966 3 860 3535.
E-mail address: mtaher@kfupm.edu.sa

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Table 1
Values of the parameters b_n, $n = 1, 3, 5, \ldots$ of Eq. (2) for the characteristic of the moderate inversion MOSFET transconductor of Eq. (1). $\Delta = 0.480453$, $D = 30.0$ and $b_n = 0$, $n = 2, 4, 6, \ldots$.

b_1	0.587665	b_{15}	−0.007362	b_{29}	0.001942
b_3	−0.21740	b_{17}	0.0055913	b_{31}	−0.00170
b_5	0.059580	b_{19}	−0.004567	b_{33}	0.001504
b_7	−0.03490	b_{21}	0.0036792	b_{35}	−0.00133
b_9	0.019586	b_{23}	−0.003106	b_{37}	0.001200
b_{11}	−0.01379	b_{25}	0.0026048	b_{39}	−0.00107
b_{13}	0.009506	b_{27}	−0.002247	b_{41}	0.000980

$2, \ldots, N$ in Eq. (2) are fitting parameters that can be obtained using the procedure described in [14,15]. This procedure is simple and does not require extensive computing facilities or well-developed software. For convenience, a brief description of this procedure is given here. First, the relationship of Eq. (1) is calculated and the offset value at $x = 0$ is removed. The resulting characteristic is normalized so that the maximum ordinate is equal to 1.0 and then mirror imaged to obtain a complete period = D. Second, this characteristic is approximated by a number of straight-line segments joined end to end. Using the slopes of these segments, it is easy to obtain the parameters b_n, $n = 1, 2, \ldots, N$ using simple algebraic calculations. Table 1 shows the resulting values. Using Eq. (2) and the parameters in Table 1, the normalized drain current was calculated and compared to Eq. (1). The results show that a relative root-mean-square (RRMS) error, given by Eq. (3), of 0.15% can be achieved. This confirms the validity of Eq. (2) for the approximating Eq. (1).

$$\text{RRMSE} = \frac{\sqrt{\sum_{i=1}^{NP}(y_{i(approximate)} - y_{i(exact)})^2}}{\sqrt{\sum_{i=1}^{NP}(y_{i(exact)})^2}} \tag{3}$$

In Eq. (3), $y_{i(exact)}$ represents the exact value of y obtained from Eq. (1), $y_{i(approximate)}$ represents the approximate value of y obtained from Eq. (2), and NP represents the total number of points used in the calculation.

3. Harmonic and intermodulation products

Eq. (2) can be used for predicting the amplitudes of the harmonics and intermodulation products of the output resulting from a normalized multi-sinusoidal gate-to-source voltage of the form

$$x(t) = X_0 + \sum_{m=1}^{M} X_m \sin \omega_m t \tag{4}$$

In Eq. (4) X_m and ω_m represent the normalized amplitude and frequency of the mth gate-to-source voltage component and X_0 represents the normalized DC bias voltage. Combining Eqs. (2) and (4) and using the trigonometric identities

$$\sin(\beta \sin \theta) = 2\sum_{k=0}^{\infty} J_{2k+1}(\beta)\sin(2k+1)\theta$$

$$\cos(\beta \sin \theta) = J_0(\beta) + 2\sum_{k=1}^{\infty} J_{2k}(\beta)\cos(2k)\theta$$

where $J_k(\beta)$ is the Bessel function of order k, and after simple mathematical manipulations, it is easy to show that the amplitude of the normalized drain current component of frequency $\sum_{m=1}^{M}\alpha_m\omega_m$ and order $\sum_{m=1}^{M}|\alpha_m|$, where α_m is a positive or negative integer or zero, will be given by

$$Y_{\alpha_1,\alpha_2,\ldots,\alpha_M} = 2\sum_{n=1}^{N} b_n \cos\left(\frac{2n\pi}{D}X_0\right)\prod_{m=1}^{M} J_{|\alpha_m|}\left(\frac{2n\pi}{D}X_m\right),$$
$$\text{for} \sum_{m=1}^{M}|\alpha_m| = \text{odd integer} \tag{5a}$$

$$Y_{\alpha_1,\alpha_2,\ldots,\alpha_M} = 2\sum_{n=1}^{N} b_n \sin\left(\frac{2n\pi}{D}X_0\right)\prod_{m=1}^{M} J_{|\alpha_m|}\left(\frac{2n\pi}{D}X_m\right),$$
$$\text{for} \sum_{m=1}^{M}|\alpha_m| = \text{even integer} \tag{5b}$$

Using Eqs. (5a) and (5b) the amplitude of the normalized drain current component of frequency ω_r, $r = 1, 2, \ldots, M$, can be expressed as

$$Y_1 = 2\sum_{n=1}^{N} b_n \cos\left(\frac{2n\pi}{D}X_0\right) J_1\left(\frac{2n\pi}{D}X_r\right) \prod_{\substack{m=1\\ m\neq r}}^{M} J_0\left(\frac{2n\pi}{D}X_m\right) \tag{6}$$

The amplitude of the kth odd-harmonic component of frequency $k\omega_r$ of the normalized drain current can be expressed as

$$Y_k = 2\sum_{n=1}^{N} b_n \cos\left(\frac{2n\pi}{D}X_0\right) J_k\left(\frac{2n\pi}{D}X_r\right) \prod_{\substack{m=1\\ m\neq r}}^{M} J_0\left(\frac{2n\pi}{D}X_m\right) \tag{7}$$

The amplitude of the kth even-harmonic component of frequency $k\omega_r$ of the normalized drain current can be expressed as

$$Y_k = 2\sum_{n=1}^{N} b_n \sin\left(\frac{2n\pi}{D}X_0\right) J_k\left(\frac{2n\pi}{D}X_r\right) \prod_{\substack{m=1\\ m\neq r}}^{M} J_0\left(\frac{2n\pi}{D}X_m\right) \tag{8}$$

The amplitude of the intermodulation product of frequency $k\omega_r \pm q\omega_s$, and order $k + q$ = odd integer, of the normalized drain current can be expressed as

$$Y_{k,q} = 2\sum_{,n=1}^{N} b_n \cos\left(\frac{2n\pi}{D}X_0\right) J_k\left(\frac{2n\pi}{D}X_r\right) J_q\left(\frac{2n\pi}{D}X_s\right) \prod_{\substack{m=1\\ m\neq r,s}}^{M} J_0\left(\frac{2n\pi}{D}X_m\right) \tag{9}$$

The amplitude of the intermodulation product of frequency $k\omega_r \pm q\omega_s$, and order $k + q$ = even integer, of the normalized drain current can be expressed as

$$Y_{k,q} = 2\sum_{,n=1}^{N} b_n \cos\left(\frac{2n\pi}{D}X_0\right) J_k\left(\frac{2n\pi}{D}X_r\right) J_q\left(\frac{2n\pi}{D}X_s\right) \prod_{\substack{m=1\\ m\neq r,s}}^{M} J_0\left(\frac{2n\pi}{D}X_m\right) \tag{10}$$

In a similar way the amplitude of any intermodulation component of any even or odd order can be obtained using Eqs. (5a) and (5b).

4. Special case

To illustrate the use of Eqs. (6)–(10), the special case of a normalized gate-to-source voltage formed of two equal-amplitude sinusoids will be considered in detail. In this case Eq. (4) reduces to

$$x(t) = X_0 + X_1(\sin\omega_1 t + \sin\omega_2 t) \tag{11}$$