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Reciprocity, fairness and learning in medium access control games

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A B S T R A C T

In wireless communication systems users compete for communication opportunities through a medium access control protocol. Previous research has shown that selfish behavior in medium access games could lead to inefficient and unfair resource allocation. We introduce a new notion of *reciprocity* in a medium access game and derive the corresponding Nash equilibrium. Further, using mechanism design we show that this type of reciprocity can remove unfair/inefficient equilibrium solutions.

The best response learning method for the reciprocity game framework is studied. It demonstrates that the game converges to the unique and stable Nash equilibrium if the nodes have low collision costs or high psychological sensitivity. For symmetric games the converged Nash equilibrium turns out to be the fair strategy.

1. Introduction

Using game theory to investigate the performance of medium access control (MAC) protocols have resulted in interesting insights. Game theory allows us to incorporate a variety of behaviors of the wireless nodes into the MAC design. One of these is the notion of reciprocity. The notion of reciprocity implies that users are neither selfish nor altruistic all the time. Rather, "they are nice to those who are nice to them, but mean to people who harm them". A corresponding property is fairness that is also dealt with in our analysis. One of the main results we derive is that reciprocity changes the best response dynamics of the game and renders bad Nash equilibria to become unstable.

A homo reciprocan model for medium access control is investigated in [1] where one notion of fairness is considered. In [2,10], the effect of compassion is studied. Game theory has been used to analyze opportunistic radio networks in [3]. Packet collisions when multiple user transmit simultaneously usually provides an incentive for selfish behavior [6]. User can selfishly cheat in a MAC game, for example, an user may not respect the random exponential backoff in CSMA/CA by adjusting the contention window arbitrarily to its minimum size. In this paper, we introduce the notion of Rabin's fairness Nash equilibrium [4] to analyze a medium access control game model with reciprocity. In our model, users have an incentive to punish selfish behaviors that allows them to avoid unfair equilibrium under certain conditions.

In [7], analysis of the fundamental equilibrium properties of the one shot random access game model is provided. It is observed that all centrally controlled optimal solutions are subsets of this game solution. This is extended in [8] to characterize the existence of uniqueness of Nash equilibria.

The transient network behavior and iterative game strategies are the focus of the studies in [9–12]. Effect of reciprocity on transient network behavior is studied in [9] using the notion of conjectural equilibrium. The operating points of the throughput region are shown to be conjecture equilibria. Dynamic altruistic behavior in Aloha random access game and its effect on altruistic behavioral stability is studied in [10].

We note that the idea of reciprocity is a "moral" correlation device that can help the medium access protocol to avoid bad equilibrium solutions. We also discuss methods to achieve good equilibrium solutions with respect to collision cost and psychological sensitivity of users. We show that an intervening mechanism designer [5] can remove the undesirable network equilibrium of MAC games by affecting the channel collision probability. The contributions of this paper are different from some of the previous works. For example, we address reciprocity in normal form MAC games while [9] addresses dynamic reciprocity. Moreover, the notion of reciprocity in [9] is different in that their addressed equilibrium is a conjectural equilibrium while we study the Nash equilibrium. One shot random access games studied in [7,8] do not consider fairness. Also unlike [8], our proposed learning mechanism converges to the unique stable Nash. Also in the symmetric game the converged Nash of our best response learning is the fair strategy while the best response in [8] does not guarantee to achieve the fair Nash.

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The paper is organized as follows. In Section 2 we discuss the standard medium access game model. Reciprocity and fairness in these games are presented in Section 3. In Section 4 a best response learning method has been presented. Simulations and numerical results are presented in Section 5. Concluding remarks are given in Section 6.

2. Standard medium access game

Let's consider a set of wireless nodes denoted by $\mathcal{I} = \{1, 2, \ldots, N\}$ contending for spectrum opportunities. If user i transmits successfully it obtains an utility u_s, if the transmission fails the utility is u_f and waiting for a spectrum opportunity leads to an utility equal to u_b s.t. $u_s > u_b > u_f$. Let the random spectrum access utility vector for user i be denoted by the vector notation $(u_{s_i}, u_{b_i}, u_{f_i})$. Then the random access game $\mathcal{G}$ is defined by the tuple $\mathcal{G} = \langle \mathcal{I},\ \{p_i\}_{i \in I},\ \{\mathbf{u}_i\}_{i \in I} \rangle$, where $p_i \in [0, 1]$ is the node transmission probability or the mixed strategy of the ith node in the game. Let us denote the channel access probability vector of the users by $\mathbf{p} = (p_1, \ldots, p_N)$. Then the game $\mathcal{G}$ is said to be in the standard form if the utility vectors are in the form of $\mathbf{u}_i = (1, 0, -\theta_i)$, where $\theta_i > 0$ is the cost of packet failure for user i.

Definition 2.1. A channel access probability $\mathbf{p}^*$ is said to be a Nash equilibrium if no node can improve its payoff by unilateral deviation, i.e. $u_i(p_i^*, p_{-i}^*) \geqslant u_i(p_i, p_{-i}^*),\ \forall p_i$.

Two games are equivalent to each other if they have the same Nash equilibria. It can be shown that any random access game is equivalent to a standard random access game by selecting $\theta_i = \frac{u_b - u_f}{u_s - u_b}$ [8]. Therefore, we have considered the standard random access games for simplicity.

2.1. Steady-state equilibrium strategies

Assume wireless nodes or players 1 and 2 are contending for the available spectrum. Then for the chosen strategies of transmit (T) or buffer (B) they receive payoffs according to the matrix M in Table 1. It can be easily shown that this game has two pure Nash equilibria, namely, $(T, B), (B, T)$ and one mixed strategy equilibrium $\left(\frac{1}{1+\theta}, \frac{1}{1+\theta}\right)$. The pure equilibria are not desirable here since in this case one user is transmitting and the other is buffering all the time. Fairness property of the mixed strategies [13] suggests the mixed strategy σ is the focal equilibrium of the random access game.

Since the probability that at least one user other than user i also transmits is given by

$$q_i(\mathbf{p}_{-i}) = 1 - \prod_{j \neq i} (1 - p_j). \tag{1}$$

the expected utility of user i playing the random access game with strategy $\mathbf{p}$ is

$$u_i(p_i, \mathbf{p}_{-i}) = p_i[-\theta_i q_i(\mathbf{p}_{-i}) + (1 - q_i(\mathbf{p}_{-i}))] \tag{2}$$

which can be written as

$$u_i(p_i, \mathbf{p}_{-i}) = p_i(1 + \theta_i)\left[\prod_{j \neq i}(1 - p_j) - \frac{\theta_i}{1 + \theta_i}\right] \tag{3}$$

$\mathbf{p}^*$ is a Nash equilibrium, if and only if for all $i \in \mathcal{I}$, the followings hold true [8]:

Table 1
Payoff matrix (M).

Strategy	Transmit	Buffer
Transmit	$-\theta, -\theta$	1, 0
Buffer	0, 1	0, 0

(i) $p_i^* = 1 \quad \text{if } \prod_{j \neq i}(1 - p_j^*) > \frac{\theta_i}{1+\theta_i}$
(ii) $p_i^* \in (0, 1) \quad \text{if } \prod_{j \neq i}(1 - p_j^*) = \frac{\theta_i}{1+\theta_i}$
(iii) $p_i^* = 0 \quad \text{if } \prod_{j \neq i}(1 - p_j^*) < \frac{\theta_i}{1+\theta_i}$

Cases (i) and (iii) indicate that following the pure Nash equilibrium strategies, depending on the value of the payoff θ_i, could lead to some nodes not transmitting at all (unfairness) or every node transmit with probability 1 (deadlock). To alleviate this problem we build the idea of reciprocity into the model. In this model, as described later, the utility function consists of two terms: (a) material and (b) psychological. We show that, reciprocity changes the best response dynamics so that bad Nash equilibria can be made unstable.

3. Fairness and reciprocity in a medium access game

Consider a two-player game in normal form (S_1, S_2, π_1, π_2) where, for $i = 1, 2,\ S_i = \{T, B\}$ is the set of actions for player i and $\pi_i(s_1, s_2)$ is the payoff for player i according to the payoff matrix M where $s_i \in S_i$ and another player j, selects $s_j \in S_j$. Let $\Delta(S_i)$ denote the set of mixed strategies of player i. For each mixed strategy vector $\boldsymbol{\sigma}_i = (\sigma_i(T),\ \sigma_i(B)) \in \Delta(S_i),\ \sum_{s_i \in \{T,B\}} \sigma_i(s_i) = 1$ where $\sigma_i(s_i)$ denotes the probability that player i will select the action $s_i \in S_i$. Then

$$\pi_i(\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_j) = \sum_{s_i \in \{T,B\}} \sum_{s_j \in \{T,B\}} \pi_i(s_i, s_j)\sigma_i(s_i)\sigma_j(s_j) \tag{4}$$

is the expected payoff for player i.

Fairness Nash formulation [4] suggests that users engage in a type of reciprocal fairness, i.e. users are neither selfish nor altruistic all the time. Rather, "they are nice to those who are nice to them, but mean to people who harm them". Therefore modelling the reciprocity of nodes in a medium access game can be done using the following model. For each $(\boldsymbol{\sigma}_j \in \Delta(S_j)$, let

$$\Pi(\boldsymbol{\sigma}_j) \equiv \{\pi_i(\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_j),\ \pi_j(\boldsymbol{\sigma}_j, \boldsymbol{\sigma}_i)\} \tag{5}$$

$\Pi(\boldsymbol{\sigma}_j)$ is the set of pair of payoffs by both players. Let values $\pi_j^h(\boldsymbol{\sigma}_j)$ and $\pi_j^l(\boldsymbol{\sigma}_j)$ denote the highest and lowest values that are Pareto efficient in $\Pi(\boldsymbol{\sigma}_j)$, and the equitable payoff is

$$\pi_j^e(\boldsymbol{\sigma}_j) = \frac{\pi_j^h(\boldsymbol{\sigma}_j) + \pi_j^l(\boldsymbol{\sigma}_j)}{2} \tag{6}$$

Let $\pi_j^{min}(\boldsymbol{\sigma}_j)$ be the worst possible payoff for player j in the set $\Pi(\boldsymbol{\sigma}_j)$. The first step to incorporate fairness into the analysis is to define a sort of "kindness" function f which measures how kind players are being to each other. From previous payoff regions, the kindness function can be defined as the following.

Definition 3.1. Suppose player i selects action $s_i \in S_i$ and believes that player j selects the mixed strategy $\boldsymbol{\sigma}_j \in \Delta(S_j)$. Then, player i's kindness to player j is

$$f_i(s_i, \boldsymbol{\sigma}_j) = \begin{cases} \frac{\pi_j(\boldsymbol{\sigma}_j, s_i) - \pi_j^e(\boldsymbol{\sigma}_j)}{\pi_j^h(\boldsymbol{\sigma}_j) - \pi_j^{min}(\boldsymbol{\sigma}_j)} & \pi_j^h(\boldsymbol{\sigma}_j) \neq \pi_j^{min}(\boldsymbol{\sigma}_j) \\ 0 & \text{Otherwise} \end{cases} \tag{7}$$

Definition 3.2. Player i's belief about how kind player j is to him given it's belief that player j is playing the mixed strategy $\boldsymbol{\sigma}_j \in \Delta(S_j)$ is given by

$$\tilde{f}_j(\boldsymbol{\sigma}_j, \boldsymbol{\sigma}_i) = \begin{cases} \frac{\pi_i(\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_j) - \pi_i^e(\boldsymbol{\sigma}_i)}{\pi_i^h(\boldsymbol{\sigma}_i) - \pi_i^{min}(\boldsymbol{\sigma}_i)} & \pi_i^h(\boldsymbol{\sigma}_i) \neq \pi_i^{min}(\boldsymbol{\sigma}_i) \\ 0 & \text{Otherwise} \end{cases} \tag{8}$$

In order to incorporate fairness into the payoffs, each user maximizes a convex combination of his material and psychological payoff as shown in (9). Suppose player i believes that player j is playing the mixed strategy $\boldsymbol{\sigma}_j \in \Delta(S_j)$ and furthermore i believes that j believes that i is playing the mixed strategy $\boldsymbol{\sigma}_i \in \Delta(S_i)$ then i's expected payoff from playing the pure strategy s_i is given by: