Computer Communications 46 (2014) 43–53

Contents lists available at ScienceDirect

Computer Communications

journal homepage: www.elsevier.com/locate/comcom

Topology management and outage optimization for multicasting over slowly fading multiple access networks ☆

Avi Zanko *, Amir Leshem, Ephraim Zehavi

School of Engineering, Bar-Ilan University, Ramat-Gan 52900, Israel

ARTICLE INFO

Article history:
Available online 21 March 2014

Keywords:
Network coding for multicasting
Wireless networks
Outage capacity
Rayleigh fading
Multiple access channels

ABSTRACT

This paper examines the problem of rate allocation for multicasting over slow Rayleigh fading channels using network coding. In the proposed model, the network is treated as a collection of Rayleigh fading multiple access channels. In this model, rate allocation scheme that is based solely on the statistics of the channels is presented. The rate allocation scheme is aimed at minimizing the outage probability. An upper bound is presented for the probability of outage in the fading multiple access channel. A sub-optimal solution based on this bound is given. A distributed primal–dual gradient algorithm is derived to solve the rate allocation problem.

1. Introduction

Network coding extends the functionality of intermediate nodes from storing/forwarding packets to performing algebraic operations on received data. If network coding is permitted, the multicast capacity of a network with a single source has been shown to be equal to the minimal min-cut between the source and each of its destinations [2]. In the past decade, the concept of combining data by network coding has been extensively extended by e.g., [3–5] and it is well known that in order to achieve the multicast rate, a linear combination over a finite field suffices if the field size is larger than the number of destinations. Moreover, centralized linear network coding can be designed in polynomial time [6]. Decentralized linear network coding can be implemented using a random code approach [7]. A comprehensive survey of network coding can be found in e.g., [8,9].

Many network resource allocation problems can be formulated as a constrained maximization of a certain utility function. The problem of network utility maximization has been explored extensively in the past few decades [10,11]. We briefly introduce related work on topology management and rate allocation for network coding in multicast over wireless networks. The problem of finding a minimum-cost scheme (while maintaining a certain multicast rate) in coded networks was studied by Lun et al. [12,13]. They showed that there is no loss of optimality when the problem is decoupled into: finding the optimal coding rate allocation vector (also known as subgraph selection) and designing the code that is applied over the optimal subgraph. Moreover, in many cases, optimal subgraphs can be found in polynomial time. If in addition the cost function is also convex and separable, the solution can be found in a decentralized manner, where message passing is required solely between directly connected nodes. This decentralized solution, if coupled with random network coding (e.g., [14,15]) provides a fully distributed scheme for multicast in coded wireline networks. This has prompted many researchers to develop different algorithms that find minimum-cost rate allocation solutions distributively; e.g., [16–19].

When addressing the problem of rate allocation for multicast with network coding in wireless networks, Lun et al. [13,20] tackled the problem through the so-called *wireless multicast advantage* phenomenon. This phenomenon simply comes down to the fact that when interference is avoided in the network (e.g., by avoiding simultaneous transmissions), communication between any two nodes is overheard by their nearby nodes due to the broadcast nature of the wireless medium. In [20], the wireless multicast advantage was used to reduce the transmission energy of the multicast scheme (since when two nodes communicate, some of their nearby nodes get the packet for "free"). Therefore, their wireline minimum-cost optimization problem was updated accordingly (see [20, Eqs. (1) and (40)]). In [19] interference is allowed but is assumed to be limited. Joint optimal power control, network coding and congestion control is presented for the case of very high SINR (signal to noise plus interference ratio). This interference

☆ This research was supported by the Israel Ministry of Labor, Trade and Commerce, as part of the RESCUE consortium. Part of this research was presented in ICCCN 2013 [1].

* Corresponding author.
E-mail addresses: avizanz@gmail.com (A. Zanko), leshem.amir2@gmail.com (A. Leshem), ephiz@yahoo.com (E. Zehavi).

http://dx.doi.org/10.1016/j.comcom.2014.03.006
0140-3664/

assumption implies that there are some limitations on simultaneous transmissions and this is taken into account in the optimization problem. In [21] the problem of joint power control, network coding and rate allocation was studied. They showed that the throughput maximization problem can be decomposed into two parts: subgraph selection at the network layer and power control at the physical layer. A primal dual algorithm was given that converges to the optimal solution provided that the capacity region is convex with respect to the power control variables (i.e., when interference are ignored). On the other hand, to take interference into account a game theoretic method was derived to approximately characterize the capacity region.

In wireless networks, it is reasonable to assume that there is no simultaneous packet transmission or reception by any transceiver. These properties of the wireless medium introduced a new cross-layer interactions that may not exist in the wired network. Sagduyu and Ephremides [22] analyzed and designed wireless network codes in conjunction with conflict-free transmission schedules in wireless ad hoc networks. They studied the cross-layer design possibilities of joint medium access control and network coding. It was shown that when certain objectives such as throughput or delay efficiency are considered, then network codes must be jointly designed with medium access control. The joint design of medium access control and network coding [22] was formulated as a nonlinear optimization problem. In [23] the work reported in [22] was extended and a linear formulation was derived.

However, there are certain other considerations that must be taken into account in the search for a rate allocation vector in wireless networks. The wireless medium varies over time and suffers from fading channels due to multipath or shadowing, for example. In [24] the block fading model was introduced. In this model the channel gain is assumed to be constant over each coherence time interval. Typically, fading models are classified as fast fading or slow fading. In fast fading, the coherence time of the channel is small relative to a code block length and as a consequence the channel is ergodic with a well-defined Shannon capacity (also known as the ergodic capacity [25]). In slow fading, the code block length and the coherence time of the channel are of the same order. Hence, the channel is not ergodic and the Shannon capacity is not usually a good measure of performance. The notion of outage capacity was introduced in [24] for transmitting over fading channels when the channel gain is available only at the receiver. In this approach, transmission takes place at a certain rate and tolerates some information loss when an outage event occurs. An outage event occurs whenever the transmitted rate is not supported by the instantaneous channel gain; i.e., when the channel gain is too low for successful decoding of the transmitted message. It is assumed that outage events occur with low probability that reliable communication is available most of the time. A different strategy to deal with slow fading is the broadcast channel approach [26]. In this approach different states of the channel are treated as channels toward different receivers (a receiver for each state). Hence, the same strategy as used for sending common and private messages to different users on the Gaussian broadcast channel can be applied here. When the channel gain is also available at the encoder, the encoder can adapt the power and the transmission rate as a function of the instantaneous state of the channel and thus can achieve a higher rate on average. Moreover, as regards the outage capacity, the transmitter can use power control to conserve power by not transmitting at all during designated outage periods.

When dealing with outage capacity for fading MAC, the common outage has a similar definition to the outage event in the point to point case. A common outage event is declared whenever we transmit with rates that are not supported by the instantaneous channel gains. If the channel gains are available at both the decoder and the encoders, additional notions of capacities for the fading MAC need to be taken into account. The throughput capacity region for the Gaussian fading MAC was introduced in [27]. In a nutshell, this is the Shannon capacity region where the codewords can be chosen as a function of the realization of the fading with arbitrarily long coding delays. However, as for the point to point case, this approach is not realistic in slow fading cases since it requires a very long delay to average out the fading effect. Hanly and Tse [28] derived the delay limited capacity for the Gaussian fading MAC (also known as the zero outage capacity). In the delay limited capacity, unlike the throughput capacity, the chosen coding delay has to work uniformly for all fading processes with a given stationary distribution. However, the delay limited capacity is somewhat pessimistic due to the demand to maintain a constant rate under any fading condition. The outage capacity region and the optimal power allocation for a fading MAC were described in [29]. As was pointed out in [29], in a slow fading environment, the decoding delay depends solely on the code-length employed and not on the time variation of the channel.

The demand for interference free channels at all nodes means that some level of orthogonality is required between different transmissions in the network. Avoiding interference between all nodes comes at the cost of loss of expensive bandwidth, or alternatively leads to rate degradation in band limited systems. The same argument can be applied to the limited interference model since some orthogonality at a certain radius is required. In [1], the MAC network coding model was introduced. In the MAC network model, in contrast to the wireless broadcast advantage based models, the superposition property of the wireless medium is exploited. The network is treated as a collection of multi access channels, such that each receiver simultaneously receives data from all its in-neighbors.

Main contributions: This paper explores the problem of rate allocation for multicasting over slow Rayleigh fading channels using network coding. The problem is examined in a model where the network is treated as a collection of Rayleigh fading multi access channels. In our network model, we assume that links on the network vary faster than the entire network can respond to the variations. Therefore, our goal is to find a rate allocation scheme that is based solely on the statistics of the channels which minimizes the outage probability. This paper differs from prior works at two major aspects. Prior works' models assume long time averaging of the instantaneous capacity (as in the ergodic capacity approach) or averaging of the packet arrival rate (see e.g., [9]). These assumptions are more suitable for fast fading model while in slow fading model this is unrealistic. Hence, in this paper we design a different rate allocation scheme which is more suitable to the slow fading model. Moreover, in this paper the design of the rate allocation scheme is based solely on the statistics which is desirable in many practical large scale networks, as will be emphasized in Section 4.

The communication model is described in detail in Section 2.

In Section 3 we present lower and upper bounds for the outage probability of a fading MAC. In Section 4 a suboptimal solution for the rate allocation problem is presented for the MAC network model. The solution is based on an upper bound on the probability of outage in the fading MAC. In Section 5 a distributed solution is derived for the rate allocation problem in the MAC network model. In Section 6 we report some simulation results. We end with concluding remarks.

2. Communication model

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a directed graph with a set of nodes $\mathcal{V}$ and directed edges $\mathcal{E} \subset \mathbf{V} \times \mathbf{V}$, where transceivers are nodes and channels are edges representing a wireless communication network. In