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Multiple scales analysis of chirped Bragg gratings

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Abstract

In this paper, a second-order multiple scales expansion is used to derive coupled-mode equations for a linearly chirped Bragg grating. This eliminates the error in the spectral response introduced by large values of the grating strength when conventional first-order coupled-mode theory is used. The autonomous and nonautonomous formulations of these equations are considered and compared in terms of accuracy and speed of the numerical solution of the resulting two-point boundary-value problem for the reflectance of the grating. These solutions are compared with the characteristic matrix solution taken as a reference. By using the fundamental matrix method, the autonomous formulation is found to be as accurate as the characteristic matrix method but faster in terms of computer CPU time.

Keywords: Optical waveguide filters; Coupled-mode theory; Chirped periodic structures; Multiple scales analysis; Bragg gratings

1. Introduction

Chirped Bragg gratings are good candidates for dispersion compensation in long-haul high-bit-rate optical fiber links [1]. These devices could be operated either in reflection mode or in transmission mode. Transmission mode compensators allow an all-fiber realization without need for circulators, thus avoiding the insertion loss of reflection mode compensators [2]. This is the main advantage of transmission mode compensators over reflection mode compensators. The latter have the advantages of larger bandwidth because of the restriction on the maximum coupling coefficient in the former so as not to produce overcoupling [3], grating length that is smaller by an order of magnitude, larger dispersion particularly when chirped, and in being less sensitive to environmental effects [4,5]. In addition, the reflection type can readily be deployed in existing networks at receiving ends. There is some prospect for apodized Bragg fiber transmission gratings [6] except that there are some technological issues that need to be resolved before they can be deployed commercially. High sensitivity to wavelength drift, considerably large grating length, inability to bend or spool, and the need to control temperature are some of these issues.

In this paper, we study the influence of grating strength and chirp parameter on chirped grating response by using the method of multiple scales [7] and employ the fundamental matrix method [8] to solve the resulting coupled-mode equations numerically. Work on apodized unchirped reflection gratings [9] using these two methods showed that the coupled-amplitude equations of coupled-mode theory were insufficiently accurate in predicting the spectral response, especially when the strength of the periodic index perturbation (δ) increases

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beyond $\delta = 0.1$. It was also shown that the perturbation method of multiple scales when extended to second order was capable of accurately predicting the response beyond $\delta = 0.2$. As in [9], the results of the present approach are compared with those obtained via the characteristic matrix method [11] that is used to verify their accuracy. Both methods are in agreement especially when the periodic refractive index profile is divided into huge number of layers. As stated in [9], the obvious advantage of the present approach over the characteristic matrix method is its computational speed. We experimented with the two different formulations of the coupled-mode equations described by McCall [10] and found that the autonomous formulation has the advantage of being faster for the same accuracy.

The thrust of the present communication is not to report on dispersion compensation but to present an appropriate model of chirping in Bragg gratings. The performance of these structures in pulse recompression has been considered in [12].

2. Formulation and multiple scales analysis

We consider a chirped grating described by a refractive index variation in the form

$$n(z) = n_a\{1 + \delta \sin[Kz + \varphi(z)]\} \tag{1}$$

where K is the wavenumber of the refractive index, δ denotes the relative fluctuation amplitude of the refractive index around the average value n_a, and $\varphi(z)$ is a slowly varying phase function describing a linear grating chirp. The problem is governed by Helmholtz's equation for a z-directed plane wave linearly polarized along the x-axis; i.e.,

$$\nabla^2 E_x + k_z^2 E_x = 0 \tag{2}$$

Here, k_z is the wavenumber given by

$$k_z = k_a(1 + \delta \sin(K_n z + \varphi(z))) \tag{3}$$

where $k_a = (\omega/c)n_a$ is the average wavenumber in the grating. The problem is governed by the following boundary conditions:

$$|E_x(z=0)| = 1 \tag{4}$$

and

$$E_x(z=L) = \frac{j\eta}{\omega\mu n_s}\frac{\partial E_x}{\partial z}\bigg|_{z=L} \tag{5}$$

where n_s is the index of refraction of the substrate at $z = L$, and η is the intrinsic wave impedance of free space.

Using the method of multiple scales, we expand E_x in powers of δ as follows:

$$E_x(z) = E_x^{(0)}(z_0, z_1, z_2) + \delta E_x^{(1)}(z_0, z_1, z_2) + \delta^2 E_x^{(2)}(z_0, z_1, z_2) + \cdots \tag{6}$$

where $z_0 = z$ is a fast varying scale, while $z_1 = \delta z$, and $z_2 = \delta^2 z$ are slowly varying independent scales. We may now express the slowly varying phase function $\varphi(z)$ in the form

$$\varphi(z_1) = F \cdot \left(\frac{z_1 - L/2}{L}\right)^2 \tag{7}$$

where F is a dimensionless chirp parameter, and L represents the grating length measured in units of the scale z_1. Substituting the above expansion for E_x into Helmholtz's equation and the boundary conditions, and equating coefficients of equal powers of δ on each side, we obtain

O(1):

$$\frac{\partial^2 E_x^{(0)}}{\partial z_0^2} + k_a^2 E_x^{(0)} = 0 \tag{8a}$$

$$|E_x^{(0)}(0)| = 1 \tag{8b}$$

$$E_x^{(0)}(L) = \frac{j\eta}{\omega\mu n_s}\frac{\partial E_x^{(0)}}{\partial z_0}\bigg|_{z_0=L} \tag{8c}$$

O(δ):

$$\frac{\partial^2 E_x^{(1)}}{\partial z_0^2} + k_a^2 E_x^{(1)} = -2\frac{\partial^2 E_x^{(0)}}{\partial z_0 \partial z_1} - 2k_a^2 \sin(Kz_0 + \varphi)E_x^{(0)} \tag{9a}$$

$$|E_x^{(1)}(0)| = 0 \tag{9b}$$

$$E_x^{(1)}(L) = \frac{j\eta}{\omega\mu n_s}\left(\frac{\partial E_x^{(1)}}{\partial z_0} + \frac{\partial E_x^{(0)}}{\partial z_1}\right)\bigg|_{z_0=L} \tag{9c}$$

O(δ^2):

$$\frac{\partial^2 E_x^{(2)}}{\partial z_0^2} + k_a^2 E_x^{(2)} = -2\frac{\partial^2 E_x^{(1)}}{\partial z_0 \partial z_1} - \frac{\partial^2 E_x^{(0)}}{\partial z_1^2} - 2\frac{\partial^2 E_x^{(0)}}{\partial z_0 \partial z_2} - 2k_a^2 \sin(Kz_0 + \varphi)E_x^{(1)} - k_a^2 \sin^2(Kz_0 + \varphi)E_x^{(0)} \tag{10}$$

3. Derivation of coupled-mode equations

In the solution of the reduced problem, two slowly varying functions representing the amplitudes of the incident and reflected waves appear. Denoting the incident and reflected amplitudes, respectively, by $A(z_1, z_2)$ and $B(z_1, z_2)$, we may write this solution in the form

$$E_x^{(0)}(z_0, z_1, z_2) = A(z_1, z_2)e^{-jk_a z_0} + B(z_1, z_2)e^{jk_a z_0} \tag{11}$$

Substituting this solution into the equations governing the O(δ) problem and using the Bragg condition

$$2k_a - K = \delta\alpha \tag{12}$$

where α is a detuning parameter measuring the nearness to resonance, secular-producing terms appear in the inhomogeneous parts. The condition for the elimination of secular