AGRICULTURAL WATER MANAGEMENT 95 (2008) 799–808

available at www.sciencedirect.com

ScienceDirect

journal homepage: www.elsevier.com/locate/agwat

Soil water movement under a single surface trickle source

P.R. Bhatnagar[a,*], H.S. Chauhan[b]

[a] Central Institute of Post Harvest Engineering and Technology, PO PAU, Ludhiana 141004, India
[b] Department of Irrigation & Drainage Engineering, G.B. Pant University of Agriculture & Technology, Pantnagar 263145, Uttarakhand, India

ARTICLE INFO

Article history:
Received 16 December 2006
Accepted 15 February 2008
Published on line 18 April 2008

Keywords:
Soil water movement
Trickle source
Oblate spheroidal coordinates
Unsaturated flow

ABSTRACT

Under a trickle source, the flow of water in unsaturated soil takes place from a disc source having a radius changing with time due to change in the rate of infiltration. To predict the wetting pattern below an emitter placed on the soil surface, an unsteady, non-linearised numerical model has been developed in an oblate spheroidal coordinate system. Using this coordinate system, the problem involving disc source geometry having radius changing with time, is simplified, as the disc is a degenerate case of an oblate spheroid. The results of the proposed model are in close agreement with the experimental results of [Taghavi, S.A., Marino, M.A., Rolston, E., 1984. Infiltration from a trickle irrigation source. J. Irrig. Drain. Eng. ASCE 110 (4), 331–341] and the numerical model of [Bresler, E., 1978. Analysis of trickle irrigation with application to design problems. Irrig. Sci. 1, 3–17] developed in cylindrical coordinates. The applicability of the model has been analysed for special conditions of trickle irrigation e.g. large time water application, redistribution of soil water after discharge is cut off or reduced, and basin irrigation with restriction of surface water flow.

1. Introduction

Trickle irrigation design parameters like emitter discharge, frequency of irrigation and spacing of the emitters, largely depend upon soil physical properties and uptake characteristics of plants under the given climatic conditions. Knowledge of the soil-wetting pattern below emitters is important for the design of a trickle irrigation system. Mathematical models have been developed to simulate soil water movement considering trickle sources as point sources (Warrick, 1974; Lockington et al., 1984; Healy and Warrick, 1988), as hemispherical sources (Ben-Asher et al., 1986; Clothier et al., 1985), or as disc sources, which is discussed below.

Surface trickle irrigation involves soil water movement from a saturated disc that is formed due to spreading of water on the soil surface till the infiltration rate matches the emitter discharge. In the course of water application, the emitter discharge may remain constant or vary, but the infiltration rate reduces with time. This results in variation in the disc radius. Therefore, under a single emitter, a three-dimensional flow of water in unsaturated soil occurs from a saturated disc having a moving boundary, i.e. the disc radius is a function of time (Brandt et al., 1971).

Numerical (Taghavi et al., 1984; Bhatnagar et al., 1997) or analytical (Warrick and Lomen, 1976) solutions of flow problems for trickle irrigation have been obtained considering a constant radius disc source. Similarly, the models developed for disc infiltrometers (Warrick, 1992) may also be extended to trickle irrigation. However, such models with constant disc radius are applicable only under special condition of large time irrigation or soil water movement after irrigation is cut off.

The steady-state solution of the three-dimensional problem for a disc source was obtained by Wooding (1968). The solution gave an asymptotic approximate expression for the total flux through the disc source, which has been widely used in studies pertaining to design of trickle irrigation (Bresler, 1978; Clothier et al., 1985). Weir (1987) derived a more accurate expression for small radii. Brandt et al. (1971) developed an

* Corresponding author. Tel.: +91 161 2301673; fax: +91 161 2301673/2308670.
E-mail address: pr_bhatn@yahoo.com (P.R. Bhatnagar).
0378-3774/$ – see front matter
doi:10.1016/j.agwat.2008.02.003

unsteady finite difference model in cartesian as well as in cylindrical coordinates for a disc source with time dependent radius. Using this model, Bresler (1978) developed procedures for estimating spacing between emitters and designing lateral systems. For the similar axi-cylindrical flow problem, Lafolie et al. (1989) presented an improved finite difference solution and compared it with that of Brandt et al. (1971) and other relevant solutions and experimental results. Moncef et al. (2002) published an expression for predicting the wetted soil volume, which was tested against finite difference solutions. All these models have been developed using conventional coordinate systems (cartesian, cylindrical, or spherical). The disc source problems involve five boundaries in such coordinate systems, making them complicated. The numerical solutions require large numbers of iterations, especially for the changing boundary conditions. An oblate spheroidal coordinate system is found to be mathematically optimal to describe the disc source problem in a simpler form, since discs can be regarded as oblate spheroids (Moon and Spencer, 1961). Philip (1985, 1989) used the oblate spheroidal coordinates to solve steady flow from disc cavities. In the present paper, a solution for unsteady flow from a disc source with a time dependent radius has been obtained to predict the soil-wetting pattern below the trickle source. The model was further extended to simulate special cases of trickle irrigation e.g. intermittent irrigation, basin irrigation, etc.

2. Problem formulation

The unsteady flow of water in unsaturated soil is formulated with the following assumptions: (i) the soil is a stable, isotropic, homogeneous and non-swelling porous medium, (ii) both the volumetric water content (θ) and the hydraulic conductivity (K) of the soil are single valued and continuous functions of the soil matric potential (h), (iii) Darcy's law is applicable in its modified form (Richards, 1931), i.e. $K = f(\theta)$, (iv) the initial water content is sufficiently low so that the movement of water is negligible everywhere in the system, and (v) evaporation and plant water extraction are negligible everywhere in the flow region.

The non-linear governing equation, derived from the continuity equation for fluid flow through a porous medium, combined with Darcy's law (Richards, 1931), is given as (z positive downward)

$$\frac{\partial\theta}{\partial t} = \nabla(K\nabla h) - \frac{\partial K}{\partial z} \tag{1}$$

Eq. (1) is simplified similar to Warrick (1974), keeping, however, the non-linearity intact. The matric flux potential (ϕ) is defined by the Kirchhoff's transformation as

$$\phi = \int_{-\infty}^{h} K(h)\,dh \tag{2}$$

Further introducing two functions, i.e. specific water capacity, $C(h) = d\theta/dh$, and the slope of the K–h curve, $s(h) = dK/dh$, in Eq. (1), and using Eq. (2), gives

$$\frac{C}{K}\frac{\partial\phi}{\partial t} = \nabla^2\phi - \frac{s}{K}\frac{\partial\phi}{\partial z} \tag{3}$$

Eq. (3) reduces to the linearised flow equation of Warrick (1974) and Warrick and Lomen (1976), considering a linear dependence of K on θ (i.e. $K = p + q\theta$, where p and q are constants) and Gardner's (1958) equation for exponential dependence of K on h as given by

$$K = K_s e^{\alpha h} \tag{4}$$

where K_s is the hydraulic conductivity at saturation and α is a constant. Eq. (3) is a non-linear partial differential equation as C, K, and s are highly non-linear functions with respect to h or ϕ.

In the present analysis, the axisymmetrical flow region is considered to be semi-infinite (i.e. $-\infty < r < \infty$, where r is the horizontal radial coordinate, and $0 < z < \infty$) with initial matric potential h_i. For the quarter flow region (Fig. 1), the initial conditions may be written as follows:

$$\phi = \phi_{\text{init}}; \quad 0 \le r < \infty; \quad 0 \le z < \infty; \quad t \le 0 \tag{5}$$

As shown in Fig. 1a, a single emitter is placed on the soil surface at a point A, and a disc of saturated soil is formed around A up to B, and the radius (r) is changing with time. Thus, the flow boundary (AC) at the soil surface ($z = 0$) requires two distinct conditions: first, the ponded circular area (AB) across which the infiltration takes place (i.e. the disc source), and second, the area outside this zone (BC). For the disc source (AB), a constant water potential boundary condition is applied, i.e. the disc remains saturated ($h = h_s$) throughout the infiltration process. As the evaporation is assumed to be negligible, no flow occurs across the soil surface in the area outside the saturated disc. Similarly, no flow occurs across the line (AD) vertically downward below the emitter representing the axis of symmetry. At large distance boundaries (CE and DE) the matric flux potential remains unaffected because the wetting front never approaches these boundaries. These boundary conditions may be written as follows:

$$\phi = \phi_s; \quad 0 \le r \le a(t); \quad z = 0; \quad t > 0 \tag{6}$$

$$K - \frac{\partial\phi}{\partial z} = 0; \quad a(t) < r < \infty; \quad z = 0; \quad t > 0 \tag{7}$$

$$\frac{\partial\phi}{\partial r} = 0; \quad r = 0; \quad 0 \le z \le \infty; \quad t > 0 \tag{8}$$

$$\lim_{r^2+z^2\to\infty} \phi = \phi_{\text{init}}; \quad t > 0 \tag{9}$$

If the emitter discharge is Q, which may or may not vary with time, the inflow through the disc source can be written as (Brandt et al., 1971)

$$Q(t) = 2\pi\int_0^{a(t)} \left[K - \frac{\partial\phi}{\partial z}\right] r\,dr \tag{10}$$

Analysis of Eq. (10) shows that, as z increases, ϕ will decrease because of increase in unsaturation. This will result in a negative value of the derivative ($\partial\phi/\partial z$). Saturation will increase with time near the disc source, which will result in reduction in the absolute value of the derivative and in turn, in