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Contact time in random walk and random waypoint: Dichotomy in tail distribution

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ABSTRACT

Contact time (or link duration) is a fundamental factor that affects performance in Mobile Ad Hoc Networks. Previous research on theoretical analysis of contact time distribution for random walk models (RW) assume that the contact events can be modeled as either consecutive random walks or direct traversals, which are two extreme cases of random walk, thus with two different conclusions. In this paper we conduct a comprehensive research on this topic in the hope of bridging the gap between the two extremes. The conclusions from the two extreme cases will result in a power-law or exponential tail in the contact time distribution, respectively. However, we show that the actual distribution will vary between the two extremes: a power-law-sub-exponential dichotomy, whose transition point depends on the average flight duration. Through simulation results we show that such conclusion also applies to random waypoint.

1. Introduction

Due to the lack of real deployments of Mobile Ad Hoc Networks (MANETs), current research on this topic is still largely based on simulation. Therefore the behavior of mobility models greatly affects simulation performance [1]. Among numerous mobility models, Random Walk (RW) and Random Waypoint (RWP) are the most widely used ones [2,3] due to their simplicity, even though many researchers have pointed out that they have many drawbacks [4–6], and proposed several new ones [7–10,5]. However, even for the simple models like RW and RWP, the relationship between their input parameters (speed, pause, flight length, flight directions, etc.) and the corresponding impact on network performance is not yet quantitatively understood.

For dense MANETs with dynamic routing protocols, network performance depends on both the mobility and the protocols. In [1] the authors proposed several protocol independent metrics including the link change rate and link duration, allowing the impact of mobility models to be evaluated through those metrics without reference to any specific protocol. For MANETs with sparser nodes, e.g., Pocket Switched Networks (PSN) [11], there are also such protocol independent metrics like the inter-contact time and the contact time [12].

In both scenarios the contact time (or alternatively, link duration, link lifetime, link expiration time, etc.[1]) has been an important performance metric in evaluating the impact of mobility. In this paper we focus on the distribution of contact times of RW and RWP. Several papers studying this distribution have been published [13–21]. These works can be divided into three categories:

1. Study using simulation or empirical data [15,19].
2. Theoretical analysis that models contact events as single direct traverses [14,16–18].
3. Theoretical analysis that models contact events as sums of multiple i.i.d. random walks [13,20,21].

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[1] All these terms refer to the same metric. In this paper the term contact time is used.

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Studies based on empirical analysis have the advantage of being accurate. In [15] the authors examined the PDFs of contact time through simulation and concluded that the PDFs are significantly different among different models. Among them the PDF of RW exhibits a single peak. The authors of [19] fit the PDF of contact time from RWP traces against several common distributions. The results showed that the lognormal distribution is the best fit for their traces.

Since it is very hard for the empirical analysis to go through all parameter spaces, theoretical derivation is necessary to better understand the underlying dynamics between the model parameters and the contact time, even though such derivation usually imposes simplifying assumptions. In RW and RWP, nodal movements are consecutive flights along straight lines. When the communication range is small in comparison to the flight length, it is reasonable to assume nodes do not stop or change directions during contact events. Thus the contact events are modeled as *direct traversals* [14,16–18]. In an early work [14] using this model, the duration distribution of two-hop paths with static sender and receiver was studied. In [16], the authors derived the contact time distribution of RW using the direct traversal model, assuming all nodes move at the same speed. In their later work [17] they extended the results with heterogeneous nodal speed. Both papers did not derive any closed form and all results were obtained numerically. In [18], the authors did a similar analysis as in [16] but derived a closed form for homogeneous speeds. They also obtained the contact time distribution numerically for two nodes with different fixed speeds.

On the other hand, when the communication range is large in comparison to the flight length, nodes often stop or change directions multiple times during contact events. Thus the contact events should then be modeled as the sum of *consecutive random walks* inside the nodes' communication range (usually modeled as circles) [13,20,21]. In an early work [13] using this model, the authors derived the probability of link availability with different initial conditions. In [20] the authors proposed a two-state Markovian framework that can be used to approximate the contact time distribution of any mobility model. They also stated that the "direct traversal" model is a special case in their framework. A comprehensive analysis of contact times using this model was done in [21], where the authors concluded that the contact time distribution can be approximated as exponential. In [21] the communication range is a random variable and the mobility model was a "smoothed" variation of RW [9]. As a special case, their conclusion also applies to ordinary RW with constant communication range.

However, both assumptions, direct traversal and consecutive random walk, are essentially two extremes in regarding the ratio of communication range and flight length. In general, the actual behavior of RW models lies in between the two extremes. In this paper we conduct a comprehensive analysis that bridge these two extreme assumptions in previous works. Especially, we investigate their difference in tail behavior. We first show that when flight lengths are infinite, which is equivalent to the direct traversal assumption, the PDF of contact time has a power-law tail with both homogeneous and uniform speed distribution. Moreover, when flight lengths are no longer infinite, the contact time distribution shows a *power-law-sub-exponential dichotomy*, with the transition point being a function of the flight time distribution. As the average flight length becomes shorter, the transition takes place earlier. When, finally, the flight length is short enough in comparison to the communication range, which is equivalent to the consecutive random walk assumption, the dichotomy degenerates into a single exponential tail, which conforms to the conclusion in [21].

The rest of the paper is organized as follows: in Section 2 the main theoretical analysis is performed. The results are validated in Section 3. Section 4 concludes the paper.

2. Model analysis

In this section the mathematical analysis of the contact time distribution is presented. In Section 2.1 we present the basic settings and assumptions. In Section 2.2 we review the general derivation of contact time distribution in [16–18] for a simplified model assuming infinite flight lengths. In Section 2.3 the power-law tail behavior is investigated assuming both homogeneous and uniform speed distribution. In Section 2.4 we consider the impact of finite flight length and reach the conclusion of the power-law-sub-exponential dichotomy.

2.1. Model settings and assumptions

2.1.1. Random walk

In the RW model all nodes take consecutive random walks along straight line segments. These walks are called "flights". For each flight a node travels in a direction ϕ at speed v for some distance u. Afterwards it pauses for some time t_{p}, and starts the next flight.

Flight parameters ϕ, v, u, and t_{p} are random variables selected independently for each flight. Also movements of different nodes are independent of each other. For most RW models, the direction ϕ is uniformly distributed over $[0, 2\pi]$, and the speed v is uniformly distributed over $[v_{\min}, v_{\max}]$ ($v_{\min} > 0$ as to obtain a stationary speed distribution [4]). The flight length u and pause time t_{p} can be fixed, or follow any common distributions like uniform, exponential or even power-law [5,22].

Throughout the analysis in this paper, all these assumptions for common RW models are used, except the pause time t_{p} is always ignored (fixed to zero). In addition, the nodal movements are confined to a taurus, which provides uniform node density and edge wrapping.

2.1.2. Random waypoint

RWP differs from RW in that for each flight every node selects a waypoint in a confined area instead of selecting the direction and the distance. Then the node travels to the waypoint through a straight line. Thus in RWP usually the node density is not uniform [23], the travel directions are not uniformly distributed, and the flight length distribution is determined by the shape of the confined area