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Exponential total variation model for noise removal, its numerical algorithms and applications

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ABSTRACT

The total variation model has been considered to be one of the most successful and representative denoising models that can preserve edges well. However, its main shortage is that it frequently causes the undesirable "block" effect. To solve this problem, high-order TV models have been proposed. Yet, they tend to damp the high frequency components in an image, often resulting in over smoothing or blurring of main features in an image. Besides, the optimization solutions underlying high-order TV models can only be obtained through numerically solving the associated high-order PDEs derived from the Euler–Lagrange equation, which is quite time-consuming. In this paper, we propose a novel total variation model based on exponential function (ETV). Furthermore, a fast numerical algorithm is designed for ETV based on Split Bregman algorithm. We test our ETV on a broad range of standard images, synthetic aperture radar (SAR) image and medical magnetic resonance images (MRI), and compared with the related TV, and high-order TV models. The experimental results have demonstrated that our ETV offers much better trade-off between noise removal and edge preservation as compared with TV and high-order TV models. In addition, ETV also shows a high computational efficiency when boosted by split Bregman algorithm.

1. Introduction

The existence of noise in digital images is inevitable due to the defects in acquisition, storage, and transmission system. And still, some other types of noise such as speckle noise are caused by some inherent law of physics during the formation of images (e.g. the speckle noise in SAR images). Noise removal has always been an important procedure in image processing, as the results of many advanced techniques, such as data structure detection [1], feature extraction, etc., are heavily determined by image quality. Therefore the noise in images must first be reduced to an acceptable level, before they can serve their purpose in any subsequent required processing.

During the past few decades, numerous methods and algorithms for noise removal have been proposed to meet the requirement for higher image quality. The total variation (TV) model which was first proposed by Rudin, Osher and Fatemi (also called ROF model) [2], is of great interest and probably the most successful one because of its simple form and decent edge preserving performance. The TV model has been widely used in image denoising, image decomposition, inpainting, deblurring, registration, etc.

Image restoration within the frame work of variation method can generally be formulated as follows:

$$\inf_u \{\beta R(u) + \varepsilon(u, f)\}, \tag{1}$$

where $R(u)$ denotes a regularization functional measuring the oscillations of an image u, while $\varepsilon(u, f)$ is a fidelity term preventing the results from getting too "far" from the original one f. $\beta > 0$ is a weight parameter balancing the importance between two terms $R(u)$ and $\varepsilon(u, f)$.

In TV model, $R(u)$is chosen as the total variation of u, and $\varepsilon(u, f)$ as an L^2-norm. TV takes the following simple form:

$$E(u) = \beta \int \left|\nabla u\right| d\Omega + \frac{1}{2}\left\|f - u\right\|_2^2, \tag{2}$$

where Ω is the image domain.

Despite the advantages TV model has, there still exist some defects and limits. For example, it frequently produces undesirable block effect. And it also shows its limitation when coping with impulsive noise and oriented textures. Many modifications have been made to improve its performance and adaptability. In our opinion, the TV has been improved in three aspects.

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In the first aspect, major efforts have been made to improve the regularization term that primarily determines the performance of image restoration. Some edge-preserving regularization models have been proposed, such as the Lorentzian potential, the Huber potential [3], and the Charbonnier potential [4]. In [5], two higher order functional models have been proposed by replacing the first order derivatives in the regularization term with two second order derivatives to reduce the block effect and improve smoothness in planar areas.

The two higher order functional are respectively given by:

$$E(u) = R_1(u) + \frac{\mu}{2}\left\|f-u\right\|_2^2 = \int (|u_{xx}| + |u_{yy}|)d\Omega + \frac{\mu}{2}\left\|f-u\right\|_2^2, \tag{3}$$

$$\begin{aligned} E(u) &= R_2(u) + \frac{\mu}{2}\left\|f-u\right\|_2^2 \\ &= \int \sqrt{|u_{xx}|^2 + \left|u_{yx}\right|^2 + \left|u_{xy}\right|^2 + \left|u_{yy}\right|^2} d\Omega + \frac{\mu}{2}\left\|f-u\right\|_2^2. \end{aligned} \tag{4}$$

Following Euler–Lagrange procedure, we obtain the following two fourth order PDEs (respectively denoted by 4OR1 and 4OR2):

$$\frac{\partial u}{\partial t} = -\left(\frac{u_{xx}}{|u_{xx}|}\right)_{xx} - \left(\frac{u_{yy}}{\left|u_{yy}\right|}\right)_{yy} + \mu(f-u), \tag{5}$$

$$\begin{aligned} \frac{\partial u}{\partial t} = &-\left(\frac{u_{xx}}{\left|D^2u\right|}\right)_{xx} - \left(\frac{u_{xy}}{\left|D^2u\right|}\right)_{yx} - \left(\frac{u_{yx}}{\left|D^2u\right|}\right)_{xy} \\ &-\left(\frac{u_{yy}}{\left|D^2u\right|}\right)_{yy} + \mu(f-u), \end{aligned} \tag{6}$$

where $\left|D^2u\right| = \sqrt{|u_{xx}|^2 + \left|u_{yx}\right|^2 + \left|u_{xy}\right|^2 + \left|u_{yy}\right|^2}$ and μ is given by:

$$\mu = \frac{1}{\sigma^2}\int\left(\frac{u_{xx}}{|u_{xx}|}(f-u)_{xx} + \frac{u_{yy}}{\left|u_{yy}\right|}(f-u)_{yy}\right)d\Omega, \tag{7}$$

with $\sigma = 1$ by default.

Also, combinations of TV and second order TV has been proposed to achieve the balance between smoothness and edge preservation [6]. In [7], a regional spatially adaptive total variation model was proposed to achieve better denoising result in flat area of images. And in [8], a shearlet based TV was proposed, in which the regularization term took good advantage of the sparse features of the shearlet representation for more effective denoising.

In the second aspect, large amount of work has been concentrated on improving the fidelity term. Fidelity term in the form of L^1-norm was proposed to suppress outliers such as salt-pepper noise [9]. Meyer proposed to replace the L^2 norm in (2) with a G norm defined in Banach space in order to capture the noise more accurately [10]. A novel texture model based on local Fourier transform has been proposed to provide a better adaptability to textures that are locally parallel and obviously oriented [11]. A TV based model with improved adaptive fidelity term (IAFT) was proposed in [12], which could improve the results in edges of images.

In the last aspect, much effort has been put into the improvement of the numerical algorithms to minimize (2). Following Euler–Lagrange procedure, the PDE corresponding to (2) was obtained:

$$\frac{\partial u}{\partial t} = \beta div\left(\frac{\nabla u}{\left|\nabla u\right|}\right) + (u_0 - u) = \frac{\beta}{\left|\nabla u\right|}\frac{\partial^2 u}{\partial T^2} + (u_0 - u), \tag{8}$$

where

$$\frac{\partial^2 u}{\partial T^2} = \frac{1}{u_x^2 + u_y^2}(u_y^2 u_{xx} - 2u_x u_y u_{xy} + u_x^2 u_{yy}), \tag{9}$$

with T denoting the tangent directions to the isophote lines.

Numerically solving this PDE with the noisy images u_0 as initial conditions, and the preset time step and number of iteration is the most straightforward way to minimize (2). However, it usually requires a large number of iteration to achieve desirable results. Many fast algorithms have been proposed, such as Chambolle's projection algorithm [13], Bregman iteration methods [14], split Bregman algorithm [15], etc. We suggest interested readers refer to [16] for a comprehensive review of the numerical algorithms and codes to minimize (2).

Let us now present the purpose of this paper. In this paper, we focus on improving the regularization term in functional (2). Although the two high-order TV models (3) and (4) can reduce block effect, they tend to damp the high frequency components in an image [17], which often results in over smoothing or blurring of main features in an image, as shown later. Moreover, until now, there is no fast algorithm to solve (3) and (4). The optimal solutions to models (3) and (4) can only be obtained by numerically solving the corresponding high-order PDEs (5) and (6), which is quite time-consuming. The purpose of this paper is to propose a novel total variation model based on exponential function. We present the development of the new method from the construction of the model to a complete, fast numerical algorithm, the choices of parameters and applications. We test our ETV on a broad range of standard test images and real-world images including one synthetic apparatus radar (SAR) image and two medical magnetic resonance images (MRI), and compare it with TV, 4OR1 and 4OR2. The experimental results have demonstrated that ETV offers much better trade-off between noise removal and edge preservation as compared with TV, and the two HOTVs. In addition, ETV takes a form as simple as TV and requires the least computational time among the four methods.

The organization of this paper is as follows. In Section 2 we describe in detail the proposed exponential total variation model and its fast numerical algorithm based on Split Bregman algorithm. In Section 3, we present experiments and comparisons. And a discussion of the experimental results is also included. Section 4 provides the summary for this paper.

2. Our ETV and its fast numerical algorithm

As mentioned above, the regularization functional primarily determines the performance of image restoration models. Unsatisfied with the results of 4OR1 and 4OR2, we restrict ourselves to proposing a new regularization functional in this section. The requirements for regularization functional are that it should be able to measure the oscillations in an image u, and be an increasing function with respect to the smoothness of u. At the same time, considering numerical algorithm of the minimization, it is highly expected that the regularization functional should take a form as simple as possible.

In the previous TV, the oscillations in an image are measured by the gradient of an image $\left|\nabla u\right|$. To obtain the stable solutions of the Eq. (2), one usually adds a small value ε to the Eq. (2), and obtains:

$$E(u) = \beta\int\sqrt{\left|\nabla u\right|^2 + \varepsilon}d\Omega + \frac{1}{2}\left\|f-u\right\|_2^2. \tag{10}$$

With the small value ε added, the original energy curve, which is a line running through origin point, becomes a hyperbolic curve whose focus point lies on the vertical axis. The hyperbolic curve intersects the vertical axis with an intercept equal to 1. And this