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Electromagnetic field in one-dimensionally anisotropic medium from a horizontal electric dipole in an isotropic region

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ABSTRACT

Electromagnetic (EM) fields in one-dimensionally anisotropic medium due to a horizontal electric dipole situated in isotropic region are investigated. The analytical expressions for the EM fields in the one-dimensionally anisotropic medium are given, and the expressions are more complex than the isotropic case, and the complete fields also include the direct field terms and the lateral wave terms. The results can be reduced to the corresponding isotropic results and proved to satisfy the boundary conditions by using the results of the EM fields in isotropic medium. The results are useful to study the propagation of the electromagnetic waves in one-dimensionally anisotropic earth or rock.

1. Introduction

The electromagnetic fields generated by a dipole located near the planar interface in stratified media like earth and air or sea water and rock or sediment have many useful applications in subsurface and closed-to-the-surface communication, radar, and geophysical prospecting and diagnostics [1–22]. Wait and King have made great contributions to this field. Wait analyzed the electromagnetic field in layered media by contour integration, and branch cuts [1–6]. King and his co-workers got the completed formulas of the electromagnetic fields for dipoles in two layered media and the summation of their work is written in the book [20], they also got the results for three layered media [18,19].

When we study the EM waves in stratified medium including the earth, the earth is not an isotropic medium and can be approximated by a one-dimensionally anisotropic half space [23–25], the properties of the electromagnetic field radiated by dipoles in this case have been investigated by several investigators [9–11]. Pan [10] had investigated the electromagnetic field when the horizontal dipole and observation point both located in the isotropic medium. In this paper, the complete formulas of the electromagnetic fields in one-dimensionally anisotropic medium due to a horizontal electric dipole in isotropic region are given, and the expressions are more complex than the corresponding isotropic case, and the complete fields also include the direct field terms and the lateral wave terms. The expressions can be easily reduced to the corresponding isotropic results. By setting $z=0$, the boundary conditions can be proved when the results of Pan in the isotropic medium are used. The results are useful to investigate the propagation of the electromagnetic waves in one-dimensionally anisotropic earth or rock. The time dependence $e^{-i\omega t}$ and the condition $|k_1^2| \gg |k_{L,T}^2|$ are used throughout the whole text.

2. Electromagnetic field in one-dimensionally anisotropic medium due to horizontal electric dipole in an isotropic region

The relevant geometry and Cartesian coordinate systems are illustrated in Fig. 1, where a unit horizontal electric dipole in the $\hat{x}$ direction is located at $(0, 0, d)$, and the observation point (ρ, ϕ, z) is in the one-dimensionally anisotropic medium. The upper half-space is Region 1 $(z \geq 0)$ occupied with isotropic medium, Region 2 $(z \leq 0)$ is the one-dimensionally anisotropic medium characterized by a permittivity tensor of the form

$$\tilde{\varepsilon}_2 = \begin{bmatrix} \varepsilon_T + i\sigma_T/\omega & 0 & 0 \\ 0 & \varepsilon_T + i\sigma_T/\omega & 0 \\ 0 & 0 & \varepsilon_L + i\sigma_L/\omega \end{bmatrix} \quad (1)$$

where ω is the angular frequency, ε_T is the transverse permittivity, and ε_L is the longitude permittivity, σ_T is the transverse conductivity, and σ_L is the longitude conductivity.

It is assumed that both Region 1 and Region 2 are non-magnetic so that their permeability are equal to μ_0, where

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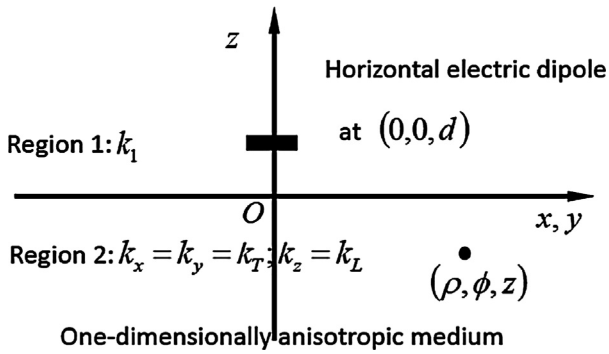

Fig. 1. The geometry and Cartesian coordinate system for a unit horizontal dipole in the isotropic medium and observation point in one-dimensionally anisotropic medium.

$\mu_0 = 4\pi \times 10^{-7}\, H/m$ is the permeability of free space. The wave numbers of the two half-spaces are

$$\begin{aligned} k_1 &= \omega\sqrt{\mu_0(\varepsilon_{1+i}\sigma_1/\omega)}, \\ k_T &= \omega\sqrt{\mu_0(\varepsilon_T + i\sigma_T/\omega)}, \\ k_L &= \omega\sqrt{\mu_0(\varepsilon_L + i\sigma_L/\omega)}, \end{aligned} \qquad (2)$$

2.1. Integral expressions for EM fields

In Pan's [10] work, formula (58) and (59) are put into formula (54) and (55), we can get

$$A_T = \frac{\eta\xi\gamma_T}{\lambda^2 M_T} e^{i\gamma_1 d} \qquad (3)$$

$$A_e = -\frac{k_T^2\gamma_1\eta\xi}{\lambda^2 N_e} e^{i\gamma_1 d} \qquad (4)$$

where η and ξ are variables of the Fourier transform, and

$$N_e = k_T^2\gamma_1 + k_1^2\gamma_e \qquad (5)$$

$$M_T = \gamma_1 + \gamma_T \qquad (6)$$

$$\begin{aligned} &\gamma_T^2 = k_T^2 - \lambda^2, Im\{\gamma_T\} > 0 \\ &\gamma_e^2 = \frac{k_T^2}{k_L^2}(k_L^2 - \lambda^2), Im\{\gamma_e\} > 0 \\ &\gamma_1^2 = k_1^2 - \lambda^2, Im\{\gamma_1\} > 0 \\ &\lambda^2 = k_x^2 + k_y^2 \end{aligned} \qquad (7)$$

By a similar way to that used by Pan [10] and King [20], the expressions of EM field in one-dimensionally anisotropic medium(Region 2) can be written as

$$\begin{aligned} E_{2\rho} = -\frac{\omega\mu_0}{4\pi k_1^2}\cos\phi \Big\{ &\int_0^\infty \frac{k_1^2}{\gamma_1}[J_0(\lambda\rho)+J_2(\lambda\rho)]e^{i\gamma_1 d}e^{-i\gamma_T z}\lambda d\lambda \\ &+2\int_0^\infty \frac{\gamma_1}{2}[J_0(\lambda\rho)-J_2(\lambda\rho)]e^{i\gamma_1 d}e^{-i\gamma_e z}\lambda d\lambda \\ &-\int_0^\infty \frac{k_1^2}{2\gamma_1}(P_T+1)[J_0(\lambda\rho)+J_2(\lambda\rho)] \times e^{i\gamma_1 d}e^{-i\gamma_T z}\lambda d\lambda \\ &+\int_0^\infty \frac{\gamma_1}{2}(Q_e-1)\times[J_0(\lambda\rho)-J_2(\lambda\rho)]e^{i\gamma_1 d}e^{-i\gamma_e z}\lambda d\lambda \Big\} \end{aligned} \qquad (8)$$

$$\begin{aligned} E_{2\phi} = \frac{\omega\mu_0}{4\pi k_1^2}\sin\phi \Big\{ &\int_0^\infty \frac{k_1^2}{\gamma_1}[J_0(\lambda\rho)-J_2(\lambda\rho)]e^{i\gamma_1 d}e^{-i\gamma_T z}\lambda d\lambda 2 \\ &\times\int_0^\infty \frac{\gamma_1}{2}[J_0(\lambda\rho)+J_2(\lambda\rho)]e^{i\gamma_1 d}e^{-i\gamma_e z}\lambda d\lambda \\ &-\int_0^\infty \frac{k_1^2}{2\gamma_1}(P_T+1)[J_0(\lambda\rho)-J_2(\lambda\rho)]e^{i\gamma_1 d}e^{-i\gamma_T z}\lambda d\lambda \\ &+\int_0^\infty \frac{\gamma_1}{2}(Q_e-1)[J_0(\lambda\rho)+J_2(\lambda\rho)]e^{i\gamma_1 d}e^{-i\gamma_e z}\lambda d\lambda \Big\} \end{aligned} \qquad (9)$$

$$E_{2z} = -\frac{i\omega\mu_0}{2\pi}\cos\phi\int_0^\infty \frac{k_T^2\gamma_1}{k_L^2 N_e}J_1(\lambda\rho)e^{i\gamma_1 d}e^{-i\gamma_e z}\lambda^2 d\lambda \qquad (10)$$

$$\begin{aligned} B_{2\rho} = \frac{\mu_0}{4\pi}\sin\phi \Big\{ &\int_0^\infty \frac{\gamma_T}{M_T}[J_0(\lambda\rho)-J_2(\lambda\rho)]e^{i\gamma_1 d}e^{-i\gamma_T z}\lambda d\lambda \\ &\times\int_0^\infty \frac{k_T^2\gamma_1}{N_e}[J_0(\lambda\rho)+J_2(\lambda\rho)]e^{i\gamma_1 d}e^{-i\gamma_e z}\lambda d\lambda \Big\} \end{aligned} \qquad (11)$$

$$\begin{aligned} B_{2\phi} = \frac{\mu_0}{4\pi}\cos\phi \Big\{ &\int_0^\infty \frac{\gamma_T}{M_T}[J_0(\lambda\rho)+J_2(\lambda\rho)]e^{i\gamma_1 d}e^{-i\gamma_T z}\lambda d\lambda \\ &\times\int_0^\infty \frac{k_T^2\gamma_1}{N_e}[J_0(\lambda\rho)-J_2(\lambda\rho)]e^{i\gamma_1 d}e^{-i\gamma_e z}\lambda d\lambda \Big\} \end{aligned} \qquad (12)$$

$$B_{2z} = \frac{i\mu_0}{2\pi}\sin\phi\int_0^\infty \frac{1}{M_T}J_1(\lambda\rho)e^{i\gamma_1 d}e^{-i\gamma_T z}\lambda^2 d\lambda \qquad (13)$$

where J_0, J_1 and J_2 are Bessel functions, and

$$Q_e = \frac{k_1^2\gamma_e - k_T^2\gamma_1}{k_1^2\gamma_e + k_T^2\gamma_1} \qquad (14)$$

$$P_T = \frac{\gamma_T - \gamma_1}{\gamma_T + \gamma_1} \qquad (15)$$

The integral formulas of the EM fields in the anisotropic medium are similar to the corresponding formulas when Region 2 is isotropic [20].

2.2. Formulation for $E_{2\rho}$ and $E_{2\phi}$

In order to evaluate the integrals of the electric fields, formula (8) and (9) can be written as

$$\begin{aligned} E_{2\rho} = -\frac{\omega\mu_0}{4\pi k_1^2}\cos\phi[&F_{\rho 01}(\rho,-z,d)+F_{\rho 02}(\rho,-z,d)+F_{\rho 11}(\rho,-z,d) \\ &+F_{\rho 12}(\rho,-z,d)] \end{aligned} \qquad (16)$$

$$\begin{aligned} E_{2\phi} = \frac{\omega\mu_0}{4\pi k_1^2}\sin\phi[&F_{\phi 01}(\phi,-z,d)+F_{\phi 02}(\phi,-z,d)+F_{\phi 11}(\phi,-z,d) \\ &+F_{\phi 12}(\phi,-z,d)] \end{aligned} \qquad (17)$$

where

$$F_{\rho 01}(\rho,-z,d) = \int_0^\infty \frac{k_1^2}{\gamma_1}[J_0(\lambda\rho)+J_2(\lambda\rho)]e^{i\gamma_1 d}e^{-i\gamma_T z}\lambda d\lambda \qquad (18)$$

$$F_{\rho 02}(\rho,-z,d) = 2\int_0^\infty \frac{\gamma_1}{2}[J_0(\lambda\rho)-J_2(\lambda\rho)]e^{i\gamma_1 d}e^{-i\gamma_e z}\lambda d\lambda \qquad (19)$$

$$F_{\rho 11}(\rho,-z,d) = -\int_0^\infty \frac{k_1^2}{2\gamma_1}(P_T+1)[J_0(\lambda\rho)+J_2(\lambda\rho)]e^{i\gamma_1 d}e^{-i\gamma_T z}\lambda d\lambda \qquad (20)$$