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Hybrid gradient-domain image denoising

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ABSTRACT

Presented is a new gradient-domain denoising method based on hybrid diffusion (thresholding) functions, combining signal gradient detection (SGD) and signal local directional variance (SLDV). In the process of denoising, the contribution of SGD and SLDV is adaptive to the contents of image. The test results presented here demonstrate that the proposed hybrid method is always on par or exceeding the current state-of-the-art gradient-domain image denoising algorithm which is named as gradient-based Wiener filter (GWF) based on SLDV and the classical Gaussian regularization anisotropic diffusion (GRAD) based on SGD, both visually and quantitatively. At the same time, the comparison compared to other reported results with related local spatial domain diffusion-based methods further verifies the good performance of the proposed method.

1. Introduction

Denoising of an image refers to the removal of noise from the observed image and remains an active research field [1]. Filtering in the gradient domain is one of the most fundamental tools for noise removal in images [2–5]. Its continuous form is termed as Perona–Malik anisotropic diffusion (PMAD) [2]. The PMAD is proposed by Perona and Malik in the 1990s. The PMAD technique has been extensively used for general image denoising in terms of preferring intra-region smoothing to inter-region diffusion since it emerged. But the PMAD has the limitation of theory and application, to cope with this problem, Catte et al. presented the Gaussian regularization anisotropic diffusion model (GRAD) [3]. Recently, Zhang et al. presented a new image denoising algorithm called gradient-based Wiener filter (GWF) [4]. At the same time, Zhang et al. show the discrete implementation of PMAD can be considered as a special GWF. The potential ability of PMAD is disclosed by investigating the relation between GWF and PMAD. More fair comparisons are presented. Although the GWF obviously outperforms the PMAD in most cases, the GWF only takes the faint advantage over PMAD with 0.10 dB in the peak signal-to-noise ratio (PSNR) for Peppers image with sharp edges. The researches show that the GRAD is poor in texture image denoising, but it also obtains good performance for the image with sharp edges. In [5], Li et al. proposed a context-adaptive anisotropic diffusion (CAAD), which still outperforms the GWF in denoising the natural image with sharp edges in term of the reported results. We note that the strategy that the product of several diffusion (thresholding) basis functions obtains a new diffusion (thresholding) function is used in CAAD. In this method, although the diffusion basis function based on spatial domain variance enlarges the range of the edges, we believe that diffusion basis function based on gradient information relieves the enlargement of the edges in the process of denoising.

To further improve the gradient-domain denoising methods, similar to strategy in [5], this paper proposes the use of hybrid diffusion (thresholding) functions combining the advantages of both SGD and SLDV, which, as far as we know, is not considered by the gradient-domain methods. The proposed method can be considered as preferring uses SGD to denoise the edges and homogeneity domains, while mainly using SLDV in detail and texture domains. The advantages of the two diffusion (thresholding) basis functions are retained in the proposed new scheme in a sense.

The paper is organized as follows. Section 2 first overviews the gradient-domain image denoising. And then two basic diffusion (thresholding) functions are analyzed. At last, the new diffusion (thresholding) function is proposed. Section 3 presents the results of experiments. The simulation results show the proposed method achieves the desired effects. Section 4 concludes the paper.

2. Hybrid gradient-domain image denoising

Assume an original image is degraded by additive noise and the noise is signal independent. The typical image degradation model at (i, j) in a two-dimensional coordinate can be written as

$$I_0(i,j) = I(i,j) + n(i,j) \tag{1}$$

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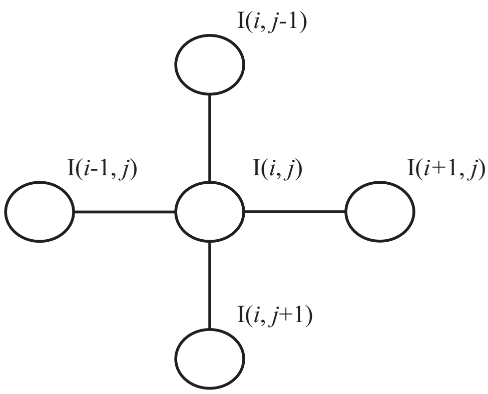


Fig. 1. The "+" window.

where I, I_0, and n represents the original image, the observed image, and the additive Gaussian random noise with zero mean and variance σ^2, respectively. Our aim is to restore the original image by using the gradient domain. In the following, we detail several respects of forming the proposed method.

2.1. *Filter in the gradient domain*

As with [4], the gradient domain for noisy image I_0 is computed as follows:

$$\nabla I_0^1(i,j) = I_0(i,j-1) - I_0(i,j) \tag{2}$$

$$\nabla I_0^2(i,j) = I_0(i,j+1) - I_0(i,j) \tag{3}$$

$$\nabla I_0^3(i,j) = I_0(i+1,j) - I_0(i,j) \tag{4}$$

$$\nabla I_0^4(i,j) = I_0(i-1,j) - I_0(i,j) \tag{5}$$

where $\nabla I_0^u(i,j)$, u = 1, 2, 3, and 4, represent the differences of four neighbors in the north, south, east and west directions centered at (i, j) in the "+" window (see Fig. 1), respectively. The $\nabla I_0^u(i,j)$ is also called the gradient coefficient for image I_0. The computations of gradient coefficient of other images are the same as that of I_0.

In the gradient domain, the shrinkage is used to get the noise gradient coefficients. The estimates $\nabla \hat{n}_0^u(i,j)$ of these noise gradient coefficients $\nabla n_0^u(i,j)$ for image I_0 is expressed as

$$\nabla \hat{n}_0^u(i,j) = \text{threshold} \cdot \nabla I_0^u(i,j) \tag{6}$$

and the computations of noise gradient coefficients of other images are the same as that of I_0.

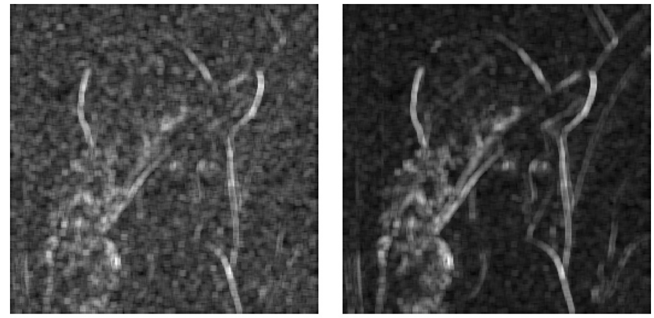
Gradient-domain algorithm is implemented by iterations. The denoising scheme can be formulated as

$$I_k(i,j) = I_{k-1}(i,j) + h \cdot \sum_{u=1}^{4} \nabla \hat{n}_{k-1}^u(i,j) \tag{7}$$

where I_k (i, j), I_0 (i, j) and $\nabla \hat{n}_{k-1}^u(i,j)$ represent the kth restored image, the original noisy image and the estimate of the gradient coefficient of the noise at the kth iteration, respectively. The parameter h is used to tune the denoising amount of each step and optimize the stopping of iterations [4].

2.2. *Two basic diffusion (thresholding) functions*

As with wavelet-based algorithm, thresholding functions play an important role in denoising performance. A lot of functions were proposed in the applications. For example, Pizurica et al. proposed a Bayesian formulation of the diffusivity function in [6]. As well as



(a) (b)

Fig. 2. The feature maps based on SLDV for Lena (256 × 256) image. (a) The map of SLDV at σ = 20; (b) the map of SLDV at σ = 10.

the related researchers proposed the diffusion functions in [2–5] and so on. Here, we focus on GWF and GRAD.

2.2.1. *GWF*

For GWF, the thresholding function is computed as follows:

$$\text{threshold} = f_1 = \frac{\sigma^2_{\nabla n^u_{k-1}}}{\sigma^2_{\nabla n^u_{k-1}} + \sigma^2_{\nabla I^u_{k-1}}(i,j)} \tag{8}$$

where $\sigma^2_{\nabla n^u_{k-1}}$ is the variance of the noise coefficients to remove in the different gradient domains, and is also called variance thresholding, which is a constant; $\sigma^2_{\nabla I^u_{k-1}}(i,j)$ is local "signal" variance in the different gradient domains, and is also called signal local directional variance (SLDV), updated by

$$\sigma^2_{\nabla I^u_{k-1}}(i,j) = \max\ (q^2_{\nabla I^u_{k-1}}(i,j) - \sigma^2_{\nabla n^u_{k-1}}, 0) \tag{9}$$

where

$$q^2_{\nabla I^u_{k-1}}(i,j) = \frac{1}{MN} \sum_{i',j' \in \omega} \nabla I^{u2}_{k-1}(i',j') - m^2_{\nabla I^u_{k-1}}(i,j) \tag{10}$$

and

$$m_{\nabla I^u_{k-1}}(i,j) = \frac{1}{MN} \sum_{i',j' \in \omega} \nabla I^u_{k-1}(i',j') \tag{11}$$

where variable ω is M-by-N local neighborhood window centered at (i, j), $q^2_{\nabla I^u_{k-1}}(i,j)$ is called local variance in the gradient domain.

In this scheme, the SLDV determines "the different regions should be preserved to what extent along the particular direction". In term of the computational way of SLDV, if the local variance at (i, j) is not greater than the variance of the noises to remove, i.e., $q^2_{\nabla I^u_{k-1}}(i,j) \le \sigma^2_{\nabla n^u_{k-1}}$, then shrinkage coefficient is 1, which shows the position (i, j) is in a "flat" region. In contrast, if the local variance is much greater than the variance of the noises to remove, which shows the location (i, j) is in a "high variance" region, and then diffusion is reduced. The form of the thresholding function is similar to Wiener filter used frequently in the wavelet domain.

Using the SLDV in the east direction, the maps of edge and texture of noisy Lena (256 × 256) images with σ = 20,10 are shown in Fig. 2. We see that the noises will be retained since the variance of noise approximates that of the textures and the edges of image (see Fig. 2a). But this case will be relieved under low noise level (see Fig. 2b). In [4], the related tests show the GWF has a good balance between the noise reduction and image feature preservation. It outperforms the classical Perona–Malik anisotropic diffusion [2], the wavelet-based Wiener filter [7], the state-of-the-art Sure-Let image denoising method [8], and so on.