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Effects of spatial receiver distributions on the user capacity of broadcast channels

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ABSTRACT

In this paper, a broadcast channel is considered in which receivers are randomly distributed on the plane. It is assumed that the x and y coordinates of receivers positions are Gaussian-distributed random variables with the same variance. Hence, the distance between each transmitter–receiver pair and consequently, the path loss term is random. It is shown that as the total number of receivers, n, in the broadcast channel goes to infinity, the maximum number of active receivers (i.e. user capacity) scales with $(\alpha/R_{\min})\ln(bn)$ where $b=(R_{\min}P^{1/\alpha})/(\alpha\sigma^2)$. Convergence of numerical results to the theoretical bounds in practical situations and the user capacity dependency on system parameters are shown in simulations results.

1. Introduction

In wireless networks, nodes are usually mobile and consequently, the distance between any transmitter–receiver pair and the path-loss term are time-varying. Due to the time-varying nature of fading channels, it is assumed that the users' channels are random over different time periods. To analyze asymptotically the throughput and the user capacity (i.e. the maximum number of users that can be activated simultaneously), fading channels statistics are required.

As the channel states become time-varying, opportunistic power allocation must optimally be performed to use system resources efficiently. In delay-sensitive applications, each user's transmission rate cannot be less than a threshold. Hence, a power allocation scheme with a minimum-rate constraint is herein considered for the receivers in the broadcast channel. As it is not always possible for all receivers to maintain this minimum rate, there is a trade-off between the total throughput and the maximum number of receivers supporting the minimum rate. To achieve the maximum total throughput, transmit power is only allocated to the receiver with the best channel gain; however, to activate more receivers, transmit power is allocated to users that can support the minimum rate and then, extra power is allocated to the receiver with the best channel gain in order to obtain a higher total throughput.

The user capacity of cellular networks in the uplink and downlink with different assumptions has been investigated in the literature. In [1], the uplink of a code-division multiple access (CDMA) cell implementing successive interference cancellation (SIC) receivers with soft decision is considered. Then, the capacity and the asymptotic user capacity (the limit when spreading gains tend to infinity with the constraint of finite system load) is obtained. In [2], the downlink user capacity of dirty paper coding (DPC) and time-division multiple access (TDMA) is compared in a single cell of a cellular network with single antennas at the transmitter and each of the receivers. Their results have been shown that TDMA may be an attractive alternative to DPC because of the complexity of the latter. In [3], the single-channel user-capacity is determined for infinite linear and planar arrays of microcells using a very idealized environment, then the best and worst capacities is computed as well as the capacity achieved by random channel placement. In [4], the user capacity of the uplink of a multiple cell synchronous CDMA system is analyzed along with power and channel allocation. For the most part, attention is focused on the situation when the signature sequences are chosen from an orthogonal sequence set. In [5], the user capacity supportable on the uplink of a multiple-macrocell CDMA system with multiple "hotspot" micro-cells embedded within is studied. It is shown that the user capacity depends on how the users are distributed among cells, and that the maximum (called the attainable capacity) occurs when all cells serve roughly the same number of users. In [6], both the uplink and downlink of a synchronous CDMA (S-CDMA) system with single cell are considered and the user capacity of a single cell with the optimal linear receiver is characterized. Finally, a power allocation scheme has been proposed in [7,8] to maximize the number of active users, for each of which, a minimum

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rate can be supported, while allocating no power to the other inactive users. As the number of supportable active users depends on the specific squared channel gains, the asymptotic behavior should be analyzed when the total number of users is large. In [7,8], under the assumption of independent Rayleigh, Rician and Nakagami fading channels for different terminals, it is shown that the maximum user capacity scales double logarithmically with the total number of users in the system and the achieving bounds are asymptotically tight.

In some applications, path loss is dominant and the effect of multipath fading can be ignored. Assuming receivers are spatially distributed on the plane, the distance between each transmitter–receiver pair is a random variable. Hence, the path loss distribution can analytically be derived. In this paper, it is assumed that path loss is dominant and the effect of spatial receiver distributions on the user capacity is asymptotically analyzed. In particular, it is assumed that the receivers are spatially distributed on the plane according to the two-dimensional Gaussian distribution. In this case, it is shown that the user capacity scales logarithmically with the total number of users in the system.

The rest of the paper is organized as follows: In Section 2, a broadcast channel model is introduced. In Section 3, assuming a spatial receiver distribution in the broadcast channel, the path-loss distribution is calculated. In Section 4, the asymptotic analysis of the user capacity is presented for Gaussian-distributed receivers in the broadcast channel. Numerical results are shown in Section 5, and the paper is concluded in Section 6.

2. System model

Consider a broadcast channel in which the transmitter is located at the origin and receivers are distributed randomly on the plane. It is assumed that the x and y coordinates of each user's position are Gaussian random variables [10]. It is also assumed that path loss is dominant and the effect of multi-path fading is neglected. Hence, the received signal for user i at time t is given by

$$R_i(t) = l_i s(t) + Z_i(t) \tag{1}$$

for $i = 1, \cdots, n$; $t = 1, \cdots, T$ where $s(t)$ is the transmitted signal, $Z_i \sim \mathcal{CN}(0, 1)$ for $i = 1, \cdots, n$ is background noise at receiver i, and l_i is the channel gain for receiver i. Note that quasi-static channels are herein considered [9]; in other words, channel gains are time-varying over different time periods but they are assumed to be constant during each time slot. Equivalently, the model (1) can be written as

$$R'_i(t) = s(t) + \frac{Z_i(t)}{l_i} \tag{2}$$

where $Z_i(t)/l_i$ is still complex Gaussian-distributed noise with the equivalent variance $N_i = 1/|l_i|^2$.

3. Path-loss distribution

As mentioned before, the x and y coordinates of each user's position are Gaussian random variables with the same variance. Hence, the distance between the transmitter and any receiver is also random. The probability density functions (pdf) of the x and y coordinates are given by

$$p_{X_i}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right) \tag{3}$$

$$p_{Y_i}(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{y^2}{2\sigma^2}\right). \tag{4}$$

Define $D_i = \sqrt{X_i^2 + Y_i^2}$ as the distance between the transmitter and receiver i, so the distance distribution is Rayleigh with the same variance as the one of the spatial receiver distribution [12]. Path loss is defined as $L = D^{-\alpha}$ where α is the path-loss exponent depending on physical characteristics of the communication channel. The cumulative distribution function (cdf) of the distance is written as

$$F_{D_i}(d) = \mathbb{P}(D_i \le d) = 1 - \exp\left(-\frac{d^2}{2\sigma^2}\right) \tag{5}$$

and consequently, the distribution of path loss is calculated as

$$\begin{aligned} F_{L_i}(l) = P(L_i \le l) &= \mathbb{P}(D_i^{-\alpha} \le l) \\ &= 1 - \mathbb{P}(D_i \le l^{-1/\alpha}) \\ &= 1 - F_{D_i}(l^{-1/\alpha}). \end{aligned} \tag{6}$$

Hence, according to (5),

$$F_{L_i}(l) = \exp\left(-\frac{l^{-2/\alpha}}{2\sigma^2}\right). \tag{7}$$

4. Asymptotic analysis of the user capacity

To achieve a trade-off between the user capacity and the total throughput, the power allocation scheme proposed in [7] is considered for receivers in the broadcast channel. In this method, to activate more receivers, transmit power is allocated to users that can support the minimum rate and then, extra power is allocated to the receiver with the best channel gain in order to obtain a higher total throughput.

In order to activate a receiver in this rate-constrained broadcast channel, its channel equivalent noise variance introduced by (2) must be less than the threshold N_0. Define M_n as the maximum number of active receivers in the broadcast channel. Then, M_n has the binomial distribution $B(n, p_0)$ [7]. For any arbitrary integer m, if $m - 1 \le np_0$ (this condition can be verified later), based on the Chernoff inequality [11],

$$P(M_n \le m - 1) \le \exp\left(-\frac{1}{2p_0}\frac{(np_0 - m + 1)^2}{n}\right). \tag{8}$$

The following optimization problem provides the maximum number of active receivers supporting the minimum required rate [7].

$$\max\{m\} \tag{9}$$

$$\ln\left(1 + \frac{P_1}{N_1}\right) \ge R_{\min} \tag{10}$$

$$\ln\left(1 + \frac{P_i}{\sum_{j=1}^{i-1} P_j + N_i}\right) = R_{\min};\ 2 \le i \le m \tag{11}$$

$$\sum_{i=1}^{m} P_i = P \tag{12}$$

where $N_i = 1/|l_i|^2$ is the equivalent variance of the noise term in (2). Without loss of generality, it is assumed that $N_1 \le N_2 \le \cdots \le N_n$. A simple algorithm solving optimization problem (9) is also proposed in [7]. It is also shown that each receiver's power is calculated by [7]

$$P_i = \frac{c}{\beta^{m-i}} \qquad i = 1, 2, \ldots, m \tag{13}$$

where m indicates the number of active receivers, $\beta = e^{R_{\min}}$ and $c = (1 - (1/\beta))P$. It is easy to show that the aforementioned power allocation satisfies the total transmit power constraint given by