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Linear prediction based adaptive algorithm for a complex sinusoidal frequency estimation

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ABSTRACT

The complex direct frequency estimation (CDFE) adaptive algorithm is developed and proposed in this paper. The motivation of this work is obtained from the previous real DFE (RDFE) adaptive algorithm. The methodology of the CDFE is based on the linear prediction property of complex sinusoidal signals. The proposed algorithm is unbiased and computationally efficient. Moreover, it is easy to implement and appropriate for real-time applications. In addition, the convergence behavior is analyzed and the steady-state mean square error (MSE) of the frequency estimate is derived in closed form. Computer simulations are treated to corroborate the theoretical analysis.

1. Introduction

Frequency estimation based on adaptive methods play a major role in many areas of digital signal processing applications [1,2] such as Doppler estimation of radar and sonar wave returns, carrier and clock synchronization, angle of arrival estimation, frequency-shift keying (FSK) demodulation and so on.

As found in the literatures, there are two groups of estimating the sinusoidal signal parameters, depending on statistical properties of the input signal, say, deterministic or random. The first one are the classical batch methods such as MUSIC [3], modified covariance (MC) [4], Pisarenko harmonic decomposition (PHD) [5], reformed PHD [6], and maximum likelihood estimation [7] which are used for estimating the sinusoidal frequency with constant in time. The last one are the sequential estimations such as the three recursive least-squares (RLS) algorithms [8] and least mean square (LMS) style algorithm [9] which are used for time varying signal frequency. In [9], So and Ching have proposed the RDFE adaptive algorithm for a real tone in noise. The concept of that work is based on the linear prediction property of real sinusoidal signals [10]. The RDFE is computationally efficient and it provides unbiased and direct frequency measurements on a sequential basis. Besides, in case of a complex sinusoidal signal frequency estimation, the batch methods [11–14] and the first-order IIR adaptive notch filter (ANF) based adaptive algorithm [15–17] can be applied. Recently, we have proposed the modified complex plain gradient (MCPG) algorithm [15]. The concept of the MCPG is based on minimizing the crosscorrelation sequence between the FIR and IIR output sequences of the ANF. It has been shown that the MCPG can improve convergence speed as compared with those of the Regalia's method (Regalia) [16] and complex plain gradient (CPG) algorithm [18] without increasing any computations. However, due to the pole contraction factor ρ of the ANF, the performances of adaptive algorithm based adaptive ANF may be poor if the selected value of ρ is inappropriate.

In this work, the linear prediction based adaptive algorithm [9] is extended to the general case of a complex sinusoidal signal. As a result, the CDFE adaptive algorithm is obtained. The CDFE is the interesting algorithm because of simplicity and computationally efficient and from the open literatures, the proposed CDFE has not been developed so far. In addition, convergence behavior of the frequency estimate in additive white Gaussian noise is analyzed and derived in terms of steady-state MSE. Moreover, the stability bound and number of iterations in which the CDFE reaches its steady-state are also obtained. Finally, computer simulations are conducted to corroborate the theoretical analysis and to demonstrate its comparative performances with the previous algorithms.

2. Problem formulation

Let us define a noisy complex sinusoid $x(k)$ of amplitude A, frequency Ω_0, and phase θ, which is of the form

$$x(k) = Ae^{j(\Omega_0 k+\theta)} + n(k), \quad k = 0, 1, 2, \ldots \tag{1}$$

Herein, $A > 0$ and $\Omega_0 \in (0, \pi)$ are considered as deterministic but unknown constants. $\theta \in [0, 2\pi)$ is a uniform random variable. $n(k) = n_R(k) + jn_I(k)$ is assumed to be a zero-mean complex white process, where $n_R(k)$ and $n_I(k)$ are zero-mean real white processes

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with identical but unknown variances of $\sigma_n^2/2$ and uncorrelated with each other. The signal to noise ratio can be calculated by

$$\mathrm{SNR} = \frac{A^2}{\sigma_n^2}. \tag{2}$$

The objective of this work is to estimate the frequency Ω_0 from the observation $x(k)$ using the linear prediction property of the following signal:

$$s(k) = e^{j\Omega_0} s(k-1). \tag{3}$$

Based on (3), the linear prediction error [14] is given by

$$e(k) = x(k) - \tilde{s}(k) = x(k) - e^{j\tilde{\Omega}_0} x(k-1), \tag{4}$$

where $\tilde{s}(k)$ is the estimate of $s(k)$, $\tilde{\Omega}_0$ is the estimate of Ω_0 and to be determined. For the batch method based frequency estimation algorithms [11–14], Ω_0 is evaluated using a block of length N of input sequence whereas for this work, it is estimated on a sample-by-sample basis. It can be shown here that the mean square value of (4) denoted by $J(\tilde{\Omega}_0)$ is (see Appendix A)

$$J(\tilde{\Omega}_0) = E[e(k)e^*(k)] = 2A^2(1 - \cos[\Omega_0 - \tilde{\Omega}_0]) + 2\sigma_n^2, \tag{5}$$

where $E[\cdot]$ is the expectation operator and the asterisk $(*)$ denotes the complex conjugation. Taking the first derivative of (5) with respect to $\tilde{\Omega}_0$ and setting the obtained result to be zero, the optimum (opt) solution is derived as

$$\tilde{\Omega}_{0,\mathrm{opt}} = \Omega_0. \tag{6}$$

In order to inspect that (6) is either global minimum or maximum point of $J(\tilde{\Omega}_0)$, the second derivative of (5) evaluated at $\tilde{\Omega}_0 = \Omega_0$ is tested and the result is

$$\left.\frac{\partial^2 J(\tilde{\Omega}_0)}{\partial \tilde{\Omega}_0^2}\right|_{\tilde{\Omega}_0 = \Omega_0} = 2A^2. \tag{7}$$

It is found that (7) is always positive for $\Omega_0 \in (0, \pi)$ and thus (6) is a global minimum of (5). Apparently, minimizing $J(\tilde{\Omega}_0)$ with respect to $\tilde{\Omega}_0$ will give the desired solution Ω_0, then unbiased frequency estimation can be attained with the use of $J(\tilde{\Omega}_0)$. In a practical point of view, the sequential based estimation algorithm is needed to minimize $J(\tilde{\Omega}_0)$. Consequently, the computationally attractive gradient-based adaptive algorithm [17] is applied to estimate Ω_0 iteratively. From (5), an instantaneous value denoted by $J(\tilde{\Omega}_0(k))$ is approximated as

$$J(\tilde{\Omega}_0(k)) = e(k)e^*(k) \tag{8}$$

where $\tilde{\Omega}_0(k)$ is the estimate of $\tilde{\Omega}_0$ at a time instant k. The stochastic gradient estimate is evaluated by differentiating $J(\tilde{\Omega}_0(k))$ with respect to $\tilde{\Omega}_0(k)$ and is given by

$$\frac{\partial J(\tilde{\Omega}_0(k))}{\partial \tilde{\Omega}_0(k)} = e(k)g^*(k) \tag{9}$$

where

$$g(k) = jx(k-1)e^{j\tilde{\Omega}_0}. \tag{10}$$

As a result, the proposed CDFE is derived as follows:

$$\tilde{\Omega}_0(k+1) = \tilde{\Omega}_0(k) + \mu Re\{e(k)g^*(k)\} \tag{11}$$

where μ is the stepsize parameter and positive constant by definition. $\mathrm{Re}\{\cdot\}$ is the real part. The derived algorithm shown in (11) is very simple and easy to implement in real-time applications. In the next section, the steady-state analysis of the proposed algorithm is addressed in terms of the closed form expression for steady-state MSE of the frequency estimate $\tilde{\Omega}_0(k)$.

3. Algorithm analysis

Prior to deriving the MSE of the frequency estimate, we evaluate the ensemble averaged value of the learning increment of (11):

$$E[\mathrm{Re}\{e(k)g^*(k)\}] = \mathrm{Re}\{E[e(k)g^*(k)]\}. \tag{12}$$

Using (1), (4) and (10) in (12), we obtain (see Appendix B)

$$E[\mathrm{Re}\{e(k)g^*(k)\}] = -A^2 \sin(\tilde{\Omega}_0 - \Omega_0). \tag{13}$$

It is obvious that $\tilde{\Omega}_0 = \Omega_0$ is a stationary point of (13). At steady-state, $\tilde{\Omega}_0 \approx \Omega_0$, the term $\sin(w)|_{w\to 0} \simeq w$ and (13) thus becomes

$$E[\mathrm{Re}\{e(k)g^*(k)\}] \approx -A^2(\tilde{\Omega}_0 - \Omega_0). \tag{14}$$

Using (14) in (11), the difference equation for the convergence in the mean of the frequency estimate is derived as follows:

$$\overline{\Omega}_0(k+1) = \overline{\Omega}_0(k) - \mu A^2(\overline{\Omega}_0(k) - \Omega_0) \tag{15}$$

where $\tilde{\Omega}_0(k)$ in (11) is replaced by $\overline{\Omega}_0(k) \equiv E[\tilde{\Omega}_0(k)]$ for notation simplicity. Eq. (15) is the first-order difference equation whose solution is given by (see Appendix C)

$$\overline{\Omega}_0(k) = (\overline{\Omega}_0(-1) - \Omega_0)(1 - \mu A^2)^{k+1} + \Omega_0 \tag{16}$$

where $\overline{\Omega}_0(-1)$ is an initial value of $\overline{\Omega}_0(k)$ at a time instant $k = -1$. It is seen that the term $(1 - \mu A^2)^{k+1}$ appeared in the right-hand side (RHS) of (16) tends to zero when $k \to \infty$ and (16) can then be rewritten as follows:

$$\overline{\Omega}_0(k)|_{k\to\infty} = \Omega_0. \tag{17}$$

Therefore it has been proved from (17) that the proposed CDFE is unbiased. Referring to (16), we can predict the iteration number N_i that is necessary to shift $\overline{\Omega}_0(k)$ from $\overline{\Omega}_0(-1)$ to Ω_0 as follows. It is well-known that the term $(1 - \mu A^2)^{k+1}$ exponentially decreases in time. Thus we can write that

$$1 - \mu A^2 = e^{-(1/\tau)} \tag{18}$$

where τ is defined as a time constant and is derived from (18) as

$$\tau = -\frac{1}{\log_e(1 - \mu A^2)}. \tag{19}$$

As a result, the convergence time N_i of the CDFE is

$$N_i \approx 5\tau \quad \text{(samples)}. \tag{20}$$

To guarantee convergence and stability, stepsize μ must be selected so that $|1 - \mu A^2| < 1$ is satisfied. Thus the bound for μ in the mean sense is easily derived as

$$0 < \mu < \frac{2}{A^2}. \tag{21}$$

From (19) and (21), some observations can be made: (i) the mean convergence rate of $\tilde{\Omega}_0(k)$ is independent of the observation noise level; (ii) it is difficult to select a value of μ to satisfy (21) when the signal amplitude A is unknown; and (iii) when A is very high, the upper bound of μ is very small, resulting in obtaining a very slow convergence rate. However, this situation hardly occurs in practice. Naturally, (21) cannot be used to discuss the stability of the algorithm since this simplified mean analysis cannot be extended to fast adaptation. Moreover, the upper bound in (21) must be scaled back due to background noise and nonstationary of the desired signal component in practice. However, this coarse boundary is useful in practice and is deserving of attention, especially when the true one is analytically infeasible.

It is also noted that the proposed CDFE does not take the form of the Regalia method [16], say, the CDFE takes the form of gradient descent procedure applied to the function $E[\mathrm{Re}\{e(k)e^*(k)\}]$ whereas the Regalia method is adjusted according to the function $E[\mathrm{Im}\{e(k)x^*(k)\}]$ (see (1), [16] where $\mathrm{Im}\{\cdot\}$ is the imaginary