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A simplified $\alpha\beta$ based Gaussian sum filter

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ABSTRACT

State estimation is a major problem in many fields, such as target tracking. For a linear Gaussian dynamic system, the KF provides the optimal state estimate, in the minimum mean square error sense. In general, however, real-world systems are governed by the presence of non-Gaussian noise and/or nonlinear systems. In this paper, the problem of state estimation in the case of a linear system affected by a non-Gaussian measurement noise is addressed. Based on the theoretical framework of the Gaussian sum filters (GSF), we propose a novel static version of this filter that uses the well known $\alpha\beta$ filter. The simulation results show that the proposed filter has acceptable performances in terms of RMSE and a reduced computational load, compared to the classical GSF.

1. Introduction

The scientific community has attached a great attention to the estimation theory in the past five decades. Many researchers have focused on the objective of obtaining a solution that can be implemented in a straightforward manner, for specific applications. The commonly used technique is the Kalman filter (KF), which gives an optimal estimation in the linear and Gaussian case. However, in practice we often have to deal with densities which, from the physics of a particular situation, are highly non-Gaussian, or densities which may have Gaussian shape in the middle, but possess small potent deviations from normality in the tails [5]. It is well known that these deviations from normality may seriously degrade the performance of a linear estimator [3]. Thus there appears to be considerable motivation for considering non-Gaussian filters [5].

The Bayesian recursion relations which describe the behavior of the a posteriori probability density function of the state of a time-discrete stochastic system, conditioned on available measurement data, cannot generally be solved in a closed-form when the system is either non-linear or non-Gaussian. In [19] a combination of Gaussian density functions is introduced and proposed as a meaningful way for approximating a non-Gaussian density. A serious limitation in this approach is that the number of Gaussian terms used to approximate the density function increases with time. After this work, an extensive research has been conducted on the Gaussian Sum Filter (GSF), which has become popular among the target tracking and localization community [9,13]. The GSF has also been used in digital transmission [8] and for nonlinear channel equalization [18]. Other approaches combine the basic GSF with other filters such as the Quadrature Kalman filter, the unscented Kalman filter, or the particle Kalman filter [11,12,14].

The fact that a non-Gaussian minimum variance filter, and specially the GSF, is considerably more difficult to implement than the Kalman filter, motivated the development of new approaches, see for example [5]. What is sought is an estimation method for non-Gaussian situations which is computationally attractive as well as easy to understand and implement [5]. Our paper is a contribution in this way.

With the advent of the Kalman filter, it was realized that the optimum time-invariant second-order Kalman filter, for tracking position and velocity, had the same architecture as the deterministic $\alpha\beta$ filter. The $\alpha\beta$ tracker is generally used for tracking targets under steady-state stationary conditions, in which the tracking problem is characterized by (a) a constant track rate, (b) a constant radar measurement noise variance, and (c) a constant target maneuverability [2]. The $\alpha\beta$ tracker has become increasingly popular, due to its simplicity and low computational load requirements. The $\alpha\beta$ tracker was used for many other applications such as: face tracking [6], intelligent vehicle [21], and recently for the problem of tracking in wireless systems [20].

In this paper the $\alpha\beta$ tracker is used within the GSF, for target tracking in the presence of a non-Gaussian measurement noise; the objective being the reduction of the complexity of the GSF.

2. State estimation of nonlinear and/or non-Gaussian measurement noise systems

In state estimation, the dynamic system is assumed to be governed by two models: (i) a process model describing the evolution of a hidden state of the system (see Eq. (1)), (ii) a noisy

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measurement model for observables, related to the hidden state (see Eq. (2)). The Bayesian framework is the most commonly used approach for estimating the state of such systems. The Bayesian approach calculates the a posteriori density of the state, obtained from Bayes' theorem, and therefore provides a complete statistical description of the state variable. At a given time k we want to estimate the state vector x_k, using the available observation vector z_k. In the Bayesian case, the a posteriori density can be determined recursively by using the previous estimate and the most recent or new measurement data, according to (3):

$$x_k = f(x_{k-1}, u_{k-1}) + v_{k-1} \tag{1}$$

$$z_k = h(x_k, u_k) + w_k \tag{2}$$

$$p(x_k|Z_k) = \frac{p(x_k|Z_{k-1})p(z_k|x_k)}{p(z_k|Z_{k-1})} \tag{3}$$

$$p(x_k|Z_{k-1}) = \int p(x_{k-1}|Z_{k-1})p(x_k|x_{k-1})dx_{k-1}, \tag{4}$$

where $Z_{k-1} = \{z_0, z_1, \ldots, z_{k-1}\}$ represents the set of all measurements up to time k–1 and the normalizing constant $p(z_k|Z_{k-1})$ in Eq. (3) is given by:

$$p(z_k|Z_{k-1}) = \int p(x_k|Z_{k-1})p(z_k|x_k)dx_k \tag{5}$$

When non-Gaussian distributions are ascribed to the initial state and/or to the noise sequences, $p(x_k|z_k)$ is no longer Gaussian and it is generally impossible to determine $p(x_k|Z_k)$ in a closed form [12].

Two main approaches have been proposed in the literature to tackle the problem of nonlinear/non-Gaussian state estimation [14]: the Gaussian Sum Filter (GSF) [19] and the particle filter [10]. The later is more computationally demanding; since it is based on Monte Carlo simulations and may require the generation and propagation of a very large sample, especially in the case of high dimensional systems [14]. Hence, in the present paper we limit our interest to the GSF, with as an aim the reduction of its complexity. The general problem, i.e. nonlinear/non-Gaussian case, was treated in [19], where the authors employ the Bayesian approach to approximate densities with a Gaussian sum, by minimizing some distance. The resulting filter could be characterized as a set of Kalman filters, whereby a set of mean values and corresponding variances are computed. The conditional state density (as well as other densities of interest) is then reconstructed as a Gaussian sum, using the aforementioned mean values and variances. As mentioned in [5], one obvious disadvantage of this method is the problem of finding an appropriate Gaussian sum representation, a task of considerable delicacy; whereas the second difficulty is that of finding some method for reducing the number of terms in the sum approximating the densities, since this number grows exponentially with the number of processed steps [5,19,23], a problem referred to as the 'growing memory problem'. A number of techniques can be used to control the number of elemental terms in a Gaussian mixture to a finite number or below a maximum number of terms [23]. Instead of carrying all the Gaussian member densities, we can either disregard some of the densities with very small mixing weights, or combine two or more similar densities. Furthermore, if needed the Gaussian mixture can be collapsed into one equivalent Gaussian term [23].

The theoretical determination of a weighted Gaussian sum approximation for a non-Gaussian density is well detailed in [19]. It can be shown that as the number of Gaussian components increases the Gaussian sum approximation converges uniformly to any probability density function [19]. We report here only the principle idea.

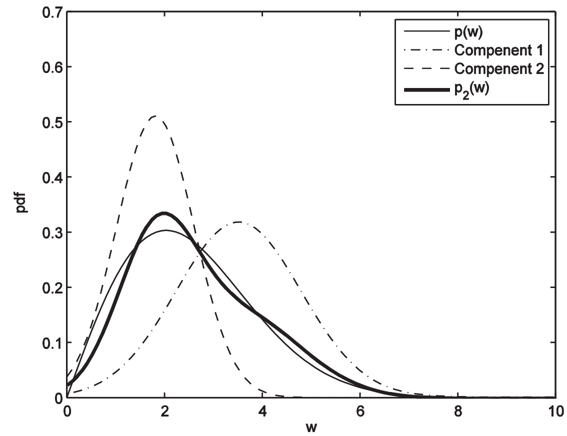

Fig. 1. Approximation of a Rayleigh distribution with two Gaussian densities.

For a given density function $p(w)$, one can have an approximation $p_m(w)$ given by:

$$p_m(w) = \sum_{i=1}^{m} \gamma_i N_{r_i}(w - \eta_i), \tag{6}$$

where $N_{r_i}(w)$ denotes a zero mean Gaussian distribution with variance r_i and where the weights γ_i satisfy:

$$\sum_{i=1}^{m} \gamma_i = 1; \quad \gamma_i = 0 \quad \text{for all} \quad i \tag{7}$$

To choose the mixture parameters, i.e. the weights γ_i, the variances r_i and the means η_i, that give the "best" approximation $p_m(w)$ to the density function $p(w)$, we have to minimize a given distance function, for example, the L^n norm [19]:

$$||p - p_m||^n = \int_{-\infty}^{+\infty} |p(x) - \sum_{i=1}^{m} \gamma_i N_{r_i}(x - \eta_i)|^n dx, \tag{8}$$

with n usually chosen equal to 2.

Alternatively, the maximum-likelihood estimates of the mixture parameters can be numerically approximated by using the Expectation Maximization (EM) algorithm [15]. An example of the approximation of a Rayleigh distribution with two Gaussian densities, using this technique, is shown in Fig. 1.

It should be mentioned here that, like in [23], in our implementation of the GSF, the construction of the Gaussian sum described above is performed once off-line, to get the Gaussian mixture parameters for the observational error that are used afterwards in the filtering procedure.

2.1. State estimation in the case of a linear system with non-Gaussian measurement noise

As it is well-known, the a posteriori, density $p(x_k|z_k)$ in (3) is Gaussian for all k when the system is linear and the initial state and plant and measurement noise sequences are Gaussian. For this case the optimal solution is given by the Kalman filter. Furthermore, in the linear Gaussian problem, the conditional mean, i.e. the minimum variance estimate, is a linear function of the measurement data and the conditional variance is independent of the measurement data. These characteristics are generally lost for a system which is either nonlinear or non-Gaussian [19].

The system noise v_k in (1) may be modeled as a non-Gaussian process, depending on the true system and how it is modeled. In fact it is introduced in the filter to account for errors modeling in