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An efficient irrigation water allocation model under uncertainty

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ABSTRACT

An interval linear fractional irrigation water allocation (ILFIWA) model is developed in response to the complexity of errors in estimating crop yields, fluctuating hydrological elements as well as varying economic profits in an irrigation system. The model is capable of quantitatively solving multi-objective problems, i.e. to obtain the maximum system net benefit and the minimum irrigation water use. Particularly, it can handle multi-objective functions expressed as ratios, such as irrigation water productivity. Moreover, it can reflect the uncertainties of the variables/parameters and functions involved. The potential of the developed model is shown by applying to a case study in northwest China. Results of the model can help make irrigation water allocation decisions for different time periods under varying flow levels. Comparison between ILFIWA model and ordinary interval linear programming model shows that the developed model is conducive to improving irrigation water productivity and saving irrigation water, and helping decision makers formulate desired irrigation water resources management policies under uncertainty.

1. Introduction

Over the past decades, the conflict between limited water resources and increased water demands has become an increasingly pressing issue due to rapid socio-economic development and continuing population growth. As the biggest consumer of limited water resources, irrigated agriculture uses about 70% of the world's freshwater withdrawals, especially in arid and semi-arid areas that are mainly characterized by low rainfall and high evaporation (García-Garizábal et al., 2011; Dai and Li, 2013). For example, in the arid area of northwest China, irrigation water consumption accounts for approximately 90% of the total water use (Huang et al., 2012). However, water shortage is subject to increasing pressure on how to reasonably and effectively allocate irrigation water resources to promote sustainable development of irrigation areas.

Growing water scarcity has led to an increasing interest in optimization modeling of irrigation water resources systems with the aim of developing and implementing appropriate water resources infrastructure and management strategies (Singh, 2012; Li et al., 2014b). Many studies on irrigation water optimal allocation have been reported in the literature (Salman et al., 2001; Sethi et al., 2006; Noory et al., 2012; Guo et al., 2014b; Li et al., 2014a). Many of the studies have focused on maximizing system benefit or minimizing water use/shortage. In reality, there is a close relationship between economic benefit and irrigation water allocation amount, especially for arid regions. If higher benefit is desired, more irrigation water will be required. Likewise, if less irrigation water use is desired, lower benefit will be attained. How to balance system benefit and irrigation water use/shortage is an issue that should be considered by decision makers. Multi-objective programming (MOP) can address such a problem (Li and Guo, 2014). In terms of the solution methods of traditional MOP for irrigation water optimal allocation, some translated certain targets into constraints, some used the method of evaluation functions, while others used goal programming or methods of optimal weights, etc. However, most of the methods mentioned above show subjective factors to a certain degree and thus affect the veracity and objectivity of results (Deb, 2014). Moreover, irrigation water productivity has become one of the indicators for water-saving high-efficiency agriculture (Fasakhodi et al., 2010). For example, decision makers in irrigation systems usually desire irrigation water allocation plans that produce maximum crop output with minimum water allocation (i.e. the maximum water-use productivity) under the situation of increasing irrigation water shortage problems. Unfortunately, the existing MOP methods do not reflect such kind of system productivity. That is, to obtain the maximum irrigation water productivity which can be expressed as irrigation net economic benefit/output (the function of irrigation water use) divides irrigation water use in a framework. Fractional programming (FP) is capable of quantitatively handling the above problem (Lotfi et al., 2010; Zhu and Huang, 2011). However, FP has been applied to irrigation systems in limited cases (Fasakhodi et al., 2010; Guo et al., 2014a).

An irrigation system is complex with many uncertainty factors, such as temporal and spatial variations of hydrological elements, fluctuations

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of economic parameters, and errors in estimating crop yields. This complexity renders the conventional optimization methods for irrigation water allocation incapable of achieving the aim of maximum system productivity. Guo et al. (2014a) developed a linear fractional programming approach for agricultural water optimal allocation considering the fuzziness of parameters. But this developed model did not express the uncertainties of decision variables and did not consider the dynamic process of water distribution. In general, specifying fuzzy sets or probability distributions is more difficult than obtaining interval numbers with known upper and lower bounds but unknown distribution information. Using interval parameters and variables is thus particularly meaningful for practical applications with the consideration of data availability and computational efficiency (Zhu, 2014). For example, in irrigation systems, the specification of crop–water production functions (CWPFs) is based on field experiment data of evapotranspiration (*ET*) and the corresponding crop yields. Nevertheless, measurement method, observation error, and computation method will directly affect the fitting results of CWPFs. Interval crop–water production functions (ICWPFs) can better reflect practical problems (Tong and Guo, 2013). Accordingly, integrating interval parameter programming (IPP) with FP is considered as a potential approach in response to obtaining the maximum irrigation water productivity under uncertainties of input variables/parameters involved. Thus far, few investigations have accounted for interval uncertainties associated with FP models in irrigation water allocation systems.

Therefore, this study aims to develop an interval linear fractional irrigation water allocation (ILFIWA) model for efficient irrigation by coupling IPP with linear fractional programming (LFP). The objective of ILFIWA model is to allocate limited irrigation water resources to different crops, obtaining maximum system net benefit with minimum water allocation. The developed model can handle irrigation water productivity problems associated with interval input parameters, avoiding the requirement of directly or indirectly setting a weight for each objective. Meanwhile, more information regarding tradeoff will be obtained. The study thus entails: (Allen et al., 1998) fitting ICWPFs by interval regression analysis method; (Chadha and Chadha, 2007) formulating ILFIWA model and developing the corresponding solution method; (Dai and Li, 2013) applying the ILFIWA model to a real case study in northwest China; and (Deb, 2014) analyzing, comparing, and discussing results.

2. Methodology

The methodology entails three major components: (Allen et al., 1998) interval crop–water production functions (Chadha and Chadha, 2007) interval linear fractional irrigation allocation model, and (Dai and Li, 2013) solution method. Each of these components is now discussed.

2.1. Interval crop–water production functions

CWPFs are used to indicate the relationship between crop yield and *ET* and constitute a suitable tool for irrigation management in water-scarcity situations (Kipkorir et al., 2002; García-Tejero et al., 2013). There are mainly two types of CWPFs: one is to describe the relationship between crop yield and *ET* in the whole growth period, while the other one is in each growth stage. The first one, including linear and nonlinear models, can be used to guide water allocation between different crops. The corresponding model expressions can be described as follows:

$$\text{Linear model}: \quad Y = c_0 ET + e_0 \tag{1a}$$

$$\text{Nonlinear model}: \quad Y = c_1 ET^2 + d_1 ET + e_1 \tag{1b}$$

where Y is the crop yield (kg/ha); ET is the evapotranspiration (m^3/ha); and c_0, e_0, c_1, d_1, e_1 are empirical coefficients, determined by regression analysis of experimental data.

The linear model is suitable for low-yield areas with insufficient irrigation water, low management level, and undeveloped agricultural resources (Kang and Cai, 1996). Hence, the linear model can be chosen to describe the relationship between crop yield and *ET* in arid and semi-arid areas. From the water balance equation, *ET* can be described as follows:

$$ET_t = M_t + P_t + K_t + \Delta H_t - F_t \tag{2}$$

where t is the time period; ET_t is the evapotranspiration in the tth period (m^3/ha); M_t is the irrigation water amount in the tth period (m^3/ha); ΔH_t is the water supply variation in the soil planned moisture layer in the tth period (m^3/ha); P_t is the effective rainfall in the tth period (m^3/ha); K_t is the groundwater recharge in the tth period (m^3/ha); and F_t is the percolation in the tth period (m^3/ha).

CWPFs are important functions that directly influence irrigation water allocation schemes. The specification of ICWPFs is necessary, considering the uncertainties, for better reflecting actual conditions. Interval regression analysis is effective to address the collected information and was chosen for obtaining ICWPFs in this study. An interval linear regression model can be written as (Montgomery et al., 2012):

$$Y(x) = A_0 + A_1(x) + \cdots + A_n(x_n) = Ax \tag{3}$$

where, $x = (1, x_1, \cdots, x_n)^T$ is a real input vector, $A = (A_0, \cdots, A_n)$ is an interval coefficient vector, and $Y(x)$ is the corresponding estimated interval. An interval coefficient A_i is denoted as $A_i = (a_i, c_i)$, where a_i is a center and c_i is a radius.

The interval regression analysis method based on quadratic programming has proven to be an effective tool to achieve the interval regression, which has advantages of giving more diverse spread coefficients than linear programming and integrating both the property of central tendency in least squares and the possibilistic property of fuzzy regression. Two parts comprise the basic formulation of interval regression by quadratic programming. The first part is $\sum_{j=1}^{m}(y_j - a^T x_j)^2$, with the meaning of the sum of squared distances between the estimated output centers and the observed outputs; the second part is $\sum_{j=1}^{m} c^T |x_j| |x_j|^T c$, representing the sum of squared spreads of the estimated outputs and $\sum_{j=1}^{m} |x_j||x_j|^T$ is a $(n+1)\times(n+1)$ symmetric positive definite matrix. The objective of the interval regression by quadratic programming is to minimize the sum of $\sum_{j=1}^{m}(y_j - a^T x_j)^2$ and $\sum_{j=1}^{m} c^T |x_j||x_j|^T c$ under different weights to determine the optimal interval coefficients $A_i = (a_i, c_i)$ $i = 0, 1, \cdots, n$. Thus, the basic formulation of interval regression by quadratic programming can be represented as follows (Tanaka and Lee, 1998):

$$\min_{a,c} J = k1 \sum_{j=1}^{m} \left(y_j - a^T x_j\right)^2 + k2 \sum_{j=1}^{m} c^T |x_j||x_j|^T c. \tag{4a}$$

To ensure that the given output y_j should be included in the estimated output $Y(x_j)$, that is, it satisfies $y_j \in Y(x_j)$, $j = 1, 2, \cdots, m$, the objective function should be subjected to the following constraints:

$$a^T x_j + c^T |x_j| \ge y_j \quad j = 1, 2, \cdots, m \tag{4b}$$

$$a^T x_j - c^T |x_j| \le y_j \quad j = 1, 2, \cdots, m \tag{4c}$$

$$c_i \ge 0 \quad i = 0, 1, \cdots, n \tag{4d}$$