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Risk programming and sparse data: how to get more reliable results

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ABSTRACT

Because relevant historical data for farms are inevitably sparse, most risk programming studies rely on few observations of uncertain crop and livestock returns. We show the instability of model solutions with few observations and discuss how to use available information to derive an appropriate multivariate distribution function that can be sampled for a more complete representation of the possible risks in risk-based models. For the particular example of a Norwegian mixed livestock and crop farm, the solution is shown to be unstable with few states of nature producing a risky solution that may be appreciably sub-optimal. However, the risk of picking a sub-optimal plan declines with increases in number of states of nature generated by Latin hypercube sampling.

1. Introduction

There are many reported studies of risk programming for farm planning. Such analyses may be performed to support a decision by an individual farmer about what farm plan to follow next year, i.e. what areas of each crop to plant and what numbers of various types of livestock to run (e.g., Olson and Eidman, 1992; Epplin and Al-Sakkaf, 1995; Pannell and Nordblom, 1998), or they may be undertaken to evaluate some proposed innovation such as a new technology or new policy instrument such as crop insurance (e.g., Krause et al., 1990; Dorward, 1999; Lien and Hardaker, 2001; Torkamani, 2005).

The form of risk programming models ranges from quadratic risk programming using a mean–variance (E,V) approximation of expected utility (Markowitz, 1952; Freund, 1956) to direct maximization of expected utility via nonlinear programming (see Hardaker et al., 2004, Chapter 9 for an overview).

A common feature of most risk programming studies is that the representation of the risk and dependency among per unit activity net revenues is based on 10 or fewer observations, typically formatted as states of nature matrices. The reason is that, in practice, the required historical data for a large number of years may not be available for the farm being analysed, or, even when the records exist, the relevance of the older information is judged to be low as the result of changed circumstances.

In regard to quadratic (E,V) risk programming applications based on sparse data, some studies have looked at the reliability of estimated optimal farm plans (e.g., Chalfant et al., 1990; Lence and Hayes, 1995) and the confidence regions in the mean–standard deviation space (e.g., Collender, 1989). Unsurprisingly, all these studies demonstrate that sparse data reduce the reliability of the risk programming results.

In this paper we use utility efficient programming (UEP) (Lambert and McCarl, 1985; Patten et al., 1988) to illustrate an approach aimed at improving reliability of results based on limited information in farm risk programming. Examples of the use of UEP for farm planning include Nanseki and Morooka (1991), Kobzar (2006) and Flaten and Lien (2007). The starting point is a sparse data set from which a multivariate probability function is specified by means of a multivariate kernel density estimation procedure. Then sampling from that distribution by two different methods is used to explore the stability and reliability of the solutions to a risk programming model. We demonstrate the approach using an example of a typical Norwegian mixed farm.

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2. The utility efficient programming model used

The UEP for the case farm was formulated as follows:

$$\max E[U] = pU(z, r), \quad (1)$$

subject to:

$$Ax \leqslant b \quad (2)$$

$$Cx + PPx - LFx - Iz = f \quad (3)$$

$$x \geqslant 0 \quad (4)$$

where, $E[U]$ is expected utility, p is a vector of probabilities for states of nature, $U(z,r)$ is a vector of utilities of net income (NI) by state of nature where the utility function is defined for a measure of risk aversion, r, A is a matrix of technical coefficients, x is a vector of activity levels, b is a vector of resource stocks, C is a matrix of gross margins, GMs, (without public payment schemes) for S states of nature, PP is a matrix of public payment schemes for S states of nature, LF is a matrix of fodder costs for livestock activities (i.e. the model finds the least-cost supply of feed) for S states of nature, I is an identity matrix, z is a vector of net incomes for each state of nature S, and f is a vector of fixed costs.

2.1. Utility and certainty equivalent

Because we assume that the farmer is risk-averse, we are restricted to using a concave form of the utility function with $U'(z) > 0$, and $U''(z) < 0$. Although in principle any kind of utility function satisfying these conditions can be used, we used the negative exponential function:

$$E[U(z, r)] = \sum_{s=1}^{S} p_s\{1 - \exp(-r \times z_s)\} \quad (5)$$

where r is a non-negative parameter representing the coefficient of absolute risk aversion with respect to net income. We assume that all states of nature are equi-probable so that $p_s = 1/S$.

This function exhibits constant absolute risk aversion (CARA), which is a reasonable approximation to the real but unknown utility function for wealth for variations in transitory (annual) income (Hardaker et al., 2004, pp 110–113). We initially assume that the farmer's relative risk aversion with respect to wealth $r_r(w) = 2$, implying moderate risk aversion. However, we do not measure utility and risk aversion in terms of wealth, but in terms of transitory income (i.e. a bad or good result in one year has little effect on wealth and hence on income levels in subsequent years). Since we use a negative exponential utility function in terms of transitory income, z, we need a relationship between $r_r(w)$ and r. Assuming asset integration, Hardaker et al. (2004, Chapter 3) show that

$$r = r_r(w)/w \quad (6)$$

The level of the farmer's wealth (net assets), w, is assumed to be NOK (Norwegian kroner) 2 million, so a value of $r = 2/2000{,}000 = 0.000001$ was used as the farmer's degree of absolute risk aversion in the main part of the analysis.

We converted the expected utility of net income of any farm plan to the corresponding certainty equivalent, CE, by taking the inverse of the utility function. While the utility function maps income to utility, the inverse function maps utility to income, i.e.

$$CE(z, r) = -\ln\{1 - E[U(z, r)]\}/r \quad (7)$$

The values of the CEs are readily interpreted because, unlike utility values, they are expressed in money terms.

To explore the effect of degree of risk aversion on the results, we later set the value of $r_r(w) = 0$ and hence $r = 0$, i.e. an extreme value implying indifference to risk. When $r = 0$, Eq. (7) is no longer relevant and the CE is equal to the expected income.

2.2. Activities and constraints

The case farm was chosen to reflect the conditions of a typical lowland farm in Eastern Norway. The main activities in the UEP model of the case farm are

(1) *Crop activities*: barley, oats, wheat, potatoes, oilseed, grass seed and carrots.
(2) *Livestock activities*: dairy cows and sheep.
(3) *Fodder crop activities*: root crops, green fodder and grassland.
(4) *Concentrate feed activities*: three different types of concentrate feed, with different levels of protein, are included. Animal feed requirements are assumed fixed per head, but choice of feed types is possible.
(5) *Hire labour and rent land activities*: provision is made to hire labour at the current wage rate of NOK 135 per hour (h). It is assumed that labour can be hired at any time of year. There is also provision to rent in land at NOK 3720 per ha, which is the average cost of renting land in Eastern Norway.
(6) *Public payment schemes*: the prevailing payment schemes (2004/2005) in Norway are included (NILF, 2005). Farmers are paid per livestock head or per ha of crop, with rates varying according to type of livestock or crop. Rates of payments decline with increases in the scale of production.

The main constraints are as follows:

(1) *Land constraint*: A farm size of 20 ha is assumed, which is close to the average farm size in the lowlands of Eastern Norway.
(2) *Rotational limits*: to avoid the build-up of pests and diseases we assume that no more than one-third of the area can be potatoes, and a maximum of one-sixth of the area can be carrots.
(3) *Marketing limit*: grass seed is regulated by production contracts. Because of the many requirements that must be satisfied to get a contract, we restrict grass seed production to 3 ha.
(4) *Milk quota constraint*: the annual milk quota of the farm is set at 100,000 l, which effectively sets an upper limit on the number of dairy cows.
(5) *Root crop limit*: because, as ruminants, cows and sheep need a minimum proportion of coarse fodder in their feed, root crops for fodder are limited to no more than 25% of the coarse fodder produced, measured in terms of livestock feed units.
(6) *Seasonal labour constraints*: there is one constraint on labour availability in each of the four seasons of spring, summer, autumn and winter. The spring season covers April and May (spring work period). The summer season is June and July, while the autumn period covers August, September and October (harvesting period). The winter season is from November to March. Labour availability is calculated on the basis of one full-time owner operator and one part-time family worker. It is assumed that the maximum amount of family labour available is 3600 h per year, distributed as 600 h in the spring and summer seasons, 900 h in the autumn season and 1500 h in the winter season. Technical input–output coefficients for seasonal labour requirements per unit of the activities are assumed fixed and are based on data from NILF (2005).
(7) *Hire labour constraint*: family labour may be supplemented with hired labour at times of peak need. The maximum amount of hired labour per year is set at 300 h, since it is sometimes difficult to get qualified farm workers in this area and unqualified workers require too much supervision.