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Spectral efficiency of random CDMA with constant envelope modulation

Ralf Müller[a,*], Alexander Lampe[b]

[a] Norwegian University of Science and Technology, 7491 Trondheim, Norway
[b] Hochschule Mittweida, 09648 Mittweida, Germany

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This paper is dedicated to Prof. Johannes Huber on the occasion of his 60th birthday.

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ABSTRACT

This paper studies the spectral efficiency of code-division multiple access with constant envelope modulation in the large system limit. Despite the fact that continuous phase modulation is hit by a severe drawback in spectral efficiency as compared to quadrature amplitude modulation on a single user communication link, the gap in terms of spectral efficiency is found to be negligible when employed on a multiple access channel with the CDMA protocol. Accounting for the efficiency of the transmit amplifier, the latter comparison becomes even favorable for constant envelope modulation.

1. Introduction

Continuous phase modulation (CPM) offers significant advantages compared to amplitude modulation with respect to the design and linearity demands of the radio-frequency (RF) hardware, in general, and of the power amplifier, in particular. These advantages made it the method of choice for the GSM (Groupe Spécial Mobile, later Global System for Mobile Communications) standard [1].

Continuous phase modulation may also bring along some disadvantages. On a single user communication link, many forms of CPM come with intrinsic intersymbol interference and the need for a trellis based equalizer at receiver. However, there are ways to circumvent this drawback in practice with negligible loss, see e.g. [2]. Nevertheless, CPM suffers from the fact that it encodes information only in the phase of the signal and does not utilize the amplitude. On a single user communication link, this ultimately results in the spectral efficiency saturating at only half its potential value at high signal-to-noise ratios (SNRs). Aiming for higher spectral efficiency the successors of GSM like IMT-2000 (International Mobile Telecommunications-2000) and others discarded CPM and went for linear modulation despite the need for more expensive RF-hardware.

The goal of this paper is to show that this was both an unlucky and also an unnecessary step. Focussing on code-division multiple-access (CDMA) which is the basis of the IMT-2000 standard, the spectral efficiency of CPM on a multiple-access channel with random spreading will be derived in the large system limit by analytical means.

With CPM using only one of the two polar components amplitude and phase, the spectral efficiency of CPM is limited to half of what is possible with quadrature amplitude modulation at high SNRs on a single-user communication link. However, in the considered multiuser system, the spectral efficiency of CPM will be found almost indistinguishable from the spectral efficiency of CDMA with linear quadrature amplitude modulation. These findings do not only hold for optimum receivers that search in the joint state-space of the modulation and the forward-error-correction (FEC) code, but also for suboptimal receivers like those specified in IMT-2000.

The paper is composed of seven more sections. Section 2 introduces the multiple-access channel considered in this paper. Section 3 introduces CDMA with continuous phase chip modulation (CPCM) as the method of choice to marry CPM and CDMA in an appropriate manner. Section 4 addresses the spectral efficiency of the proposed constant envelope CDMA. Section 5 proposes spectral shaping to improve the spectral efficiency of CPCM and Section 6 compares the system performance accounting for losses by heat dissipation in the transmit amplifiers. Section 7 addresses the issue of building a receiver with moderate complexity. Conclusions are drawn in Section 8.

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[*] Corresponding author. Tel.: +47 73594417; fax: +47 73592640.
E-mail address: ralf@iet.ntnu.no (R. Müller).

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2. System model

Consider an asynchronous CDMA system with K users in flat[1] Rayleigh fading. We focus on the uplink (reverse link) communication here, since the choice of CPM is motivated the by cost of transmitter hardware which is more relevant for mobile devices than for base stations.

The received signal at the base station is given by

$$y(t)=\sum_{k=1}^{K} a_k s_k(t-\tau_k)+w(t) \tag{1}$$

in complex base-band notation. Here, $a_k \in \mathbb{C}$ is the received amplitude of user k accumulating the effects of transmitted amplitude, fading, and carrier phase offset; $\tau_k \in \mathbb{R}$ is the signal delay of user k; $w(t)$ is zero-mean additive white Gaussian noise with power spectral density N_0; and $s_k(t)$ is the spread signal of user k. It can be decomposed into

$$s_k(t)=\sum_{m=-\infty}^{+\infty} b_k[m] c_k(t-mT_s) \tag{2}$$

where $b_k[m]$ is the symbol transmitted by user k at time instant m, $c_k(t)$ is the symbol waveform, and T_s is the symbol clock cycle. Introducing the spreading factor N and the chip clock cycle

$$T_\mathrm{c}=\frac{T_\mathrm{s}}{N} \tag{3}$$

we represent the symbol waveform

$$c_k(t)=\sum_{n=0}^{N-1} s_k[n]\psi(t-nT_\mathrm{c}) \tag{4}$$

by the discrete-time chip sequence $s_k[\cdot]$ and the chip waveform $\psi(t)$. Finally, we assume that $\tau_k \bmod T_\mathrm{c}$ is uniformly distributed within the chip interval.

3. Continuous phase chip modulation

The combination of CPM and CDMA was first addressed in [3] in a very general setting. In the sequel, the following two ways to combine CPM with CDMA were given further attention. One the one hand, we can generate a traditional spread spectrum signal and feed it into a CPM modulator as proposed in [4]. This allows for a simple implementation of the transmitter by means of a voltage controlled oscillator. However, it prohibits linear multiuser detection, as it does not allow for a description of the transmitted signal as linear or quasi-linear CDMA unless the received signal can be closely approximated by an offset quaternary phase-shift keying (OQPSK) signal as proposed in [5]. One the other hand, we can pass the spreading sequence through a CPM modulator before we modulate the data sequence with it as proposed in [6–8]. This method decouples the phase state of the CPM modulator from the phase of the data signal and therefore constitutes a linear CDMA system with constant envelope. In the sequel, it will be referred to as *continuous phase chip modulation* (*CPCM*).

Without lack of generality, CPCM can be represented by (2) where the symbol waveform $c_k(t)$ is a CPM signal of approximate duration T_s and approximate bandwidth $1/T_\mathrm{c}$. Due to the multiplication with the data sequence, phase jumps can occur at time instances that are integer multiples of T_s, unless the symbol waveform is chosen appropriately. In order to avoid those phase jumps, we restrict the spreading factor N to be even, the data sequence to be real and binary, i.e. $b_k[\cdot] \in \{\pm 1\}$, and the spreading sequences and the symbol waveforms to obey decomposition (4) with

$$j^n s_k[n] \in \{\pm 1\} \tag{5}$$

and

$$\psi(t)=\sin(2\pi h q(t)), \tag{6}$$

respectively, where $h+1/2 \in \mathbb{Z}$,

$$q(t)=\frac{1}{2}\int_{-\infty}^{t} f\left(t'+\frac{T_\mathrm{c}}{2}\right)+f\left(t'-\frac{T_\mathrm{c}}{2}\right)\,\mathrm{d}t', \tag{7}$$

and the frequency pulse shape $f(t)$ obeys

$$f(t)=f(-t) \tag{8}$$

$$f(t)=0 \quad \forall |t| \geq \frac{T_\mathrm{c}}{2} \tag{9}$$

$$\int_{-\infty}^{+\infty} f(t)dt=1. \tag{10}$$

This representation makes use of the fact that certain CPM signals can be expressed as OQPSK signals. However, here the offset is not $T_\mathrm{c}/2$, but T_c, resulting in binary data and a duplication of the frequency pulse (7). The ideas behind (5) to (10) are as follows: (5) ensures that even and odd chips are mapped to different quadrature components; (6) ensures the constant envelope by means of the well-known identity $\sin^2(x)+\cos^2(x)=1 \forall x$ provided that chips mapped to the same quadrature component do not overlap; the avoidance of such overlap and of phase jumps between chips is ensured by (7) to (10).

It will improve the shape of the power spectral density if $f(t)$ is a smooth function, though this is not strictly required for the validity of the considerations to follow.

4. Spectral efficiency

CPCM only allows for real-valued data sequences. On a single user channel, this would jeopardize spectral efficiency since the quadrature component would stay unused. However, on a multiple-access channel, the quadrature component is utilized by other users whose carrier phase is appropriately shifted. Utilizing the concept of widely linear detection [9], it was shown [10,11] that restricting to real-valued data symbols does not affect the spectral efficiency of CDMA in the large system limit.

While we should not be concerned with the restriction to real-valued modulation in respect of [10,11], it is also evident from (5) that CPCM imposes a severe restriction to the choice of the spreading sequences. This raises the question whether this restriction reduces the space of available spreading sequences in such a way that spectral efficiency will degrade. In order to debunk this concern consider two CPCM users, say k and k', who are shifted in time by T_c and shifted in phase by $\pi/2$ and employ random spreading such that $j^n s_k[n]$ and $j^n s_{k'}[n]$ are independent identically distributed (i.i.d.) sequences. Their superimposed signal follows the same statistics[2] as the signal of a single user with complex Gray-mapped data symbols and complex-valued i.i.d. random chips utilizing the same chip pulse $\psi(t)$. Both cases obtain identical spectral efficiency and noise resistance. Furthermore, for each user k there will exist such a matched partner user k' in the large system limit due to the law of large numbers. Thus, without loss of generality, we will consider

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[1] We restrict the consideration to flat fading for sake of simplicity. The effect of multipath can be easily incorporated into the shape of the chip waveform if the excess delay is much shorter than the symbol interval.

[2] Since we are considering a large system with $N \to \infty$, we can neglect the edge effects occurring for the first and the last chip within a symbol interval.