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Global stability of Peer-to-Peer file sharing systems

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Abstract

A lot of recent research has been focusing on the steady state performance of Peer-to-Peer (P2P) networks. However, it is not yet clear to us whether the steady state of a P2P system always exists. In this paper, we study the stability issue of P2P file sharing systems. We use a simple fluid model to analyze BitTorrent-like P2P file sharing networks. The resulting fluid model is a switched linear system and we prove that such a system is always globally stable. We also provide numerical results based on extensive simulations.

Keywords: Peer-to-Peer networks; Fluid model; Global stability

1. Introduction

In recent years, many P2P applications, from traditional file sharing to P2P live audio/video streaming, have been developed. Among these applications, P2P file sharing remains one of the most popular applications and its traffic has been dominating the Internet bandwidth for the past years. There are also many P2P file sharing applications, like Gnutella, Kazaa, eDonkey, BitTorrent [1], etc.

In P2P file sharing, an interested file is divided into many pieces. The size of each piece ranges from several hundred kilobytes to several megabytes. When a new peer joins the network, it begins to download pieces from other peers. As long as it obtains one piece of the file, the new peer can start to serve other peers by uploading pieces. Since peers are downloading and uploading at the same time, when the network becomes large, although the demands increase, the service provided by the network also increases. Hence, the performance of the P2P network scales very well.

BitTorrent has been one of the most popular P2P file sharing applications and has attracted a lot of research attentions. While early work on P2P systems has mainly focused on system design and traffic measurements, [2–4], some recent research has emphasized performance modeling. In [5], a closed queueing system is used to model a general P2P file sharing system and basic insights on the stationary performance are provided. In [6,7], a stochastic fluid model is used to study the performance of P2P web cache (SQUIRREL) and cache clusters. In [8,9], a branching process is used to study the service capacity of BitTorrent-like P2P file sharing in the transient regime and a simple Markovian model is presented to study the steady state properties. In [10], a spatio-temporal model is proposed to analyze the resource usage of P2P systems. In [11], an approximation for the life time of a chunk in BitTorrent is proposed. Reference [12] presents an extensive trace analysis and modeling study of BitTorrent-like systems. In [13], the authors studied the behavior of peers in BitTorrent and also investigated the file availability and the dying-out process. In [14], we proposed simple fluid models to study the performance and scalability of BitTorrent-like P2P systems.

From real trace measurements, it has been observed that a BitTorrent-like P2P file sharing network has three phases [8], a growing phase, a stabilizing phase, and a decaying phase. In the stabilizing phase, the system enters a “steady state”, in which the number of peers and the performance of each peer are relatively stable. The stabilizing phase is normally the one that most of the downloads take place

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and hence it has significant impact on the system performance. A lot of research has been done on the performance analysis in the stabilizing phase. However, theoretically, it is still not clear to us whether a P2P file sharing system always has a stabilizing phase. And if not, what are the conditions required for the system to be stable. For example, if the number of peers in the system never enters a steady state and keeps oscillating, it will be meaningless to analyze the steady state performance. Hence, the stability of a P2P system is a fundamental problem for any serious P2P performance analysis and is the question we would like to answer in this paper. As far as we know, this is the first work to theoretically analyze the global stability of BitTorrent-like P2P networks. Note that there are some work in the literature on the stability of P2P networks. However, the stability studied in those work are not the same one we studied here. For example, in [15], the stability of Chord-based P2P systems is studied. The stability there, however, is mainly about how stable or robust the overlay network connections can be maintained if there are peers leaving the system. While in our paper, we are interested in whether the system will ever enter into a steady state, in which we can then analyze the performance such as the average download rate of each peer, etc.

The paper is organized as follows. In Section 2, we will briefly review the fluid model and introduce the stability problem. In Section 3, we study the local stability of the fluid model and in Section 4, we prove that the given model is globally stable. We will then verify our results by simulations in Section 5. Finally, we will conclude this paper in Section 6.

2. Fluid model

In this paper, we choose the fluid model that has been proposed in [14] to study the stability of BitTorrent-like P2P systems. In recent years, fluid model has been used to study the performance of many P2P systems [6,7,14]. When the system is very large, which is normally the case in P2P networks, fluid model can very accurately capture the network behavior. It also has the merits of simplicity and hence is mathematically tractable. Next, we give a brief overview of the BitTorrent fluid model.

BitTorrent distinguishes between two types of peers, namely *downloaders* and *seeds*. Downloaders are peers who only have a part (or none) of the file while seeds are peers who have all the pieces of the file but stay in the system to allow other peers to download from them. Thus, seeds only perform uploading while downloaders download pieces that they do not have and upload pieces that they have. In the fluid model, we use the following quantities to capture a BitTorrent-like P2P network that serves a given file (without loss of generality, we assume that the file size is 1):

$x(t)$ number of downloaders (also known as leechers) in the system at time t.
$y(t)$ number of seeds in the system at time t.
$\lambda > 0$ the arrival rate of new peers.
$\mu > 0$ the uploading bandwidth of a given peer. We assume that all peers have the same uploading bandwidth.
$c > 0$ the downloading bandwidth of a given peer. We assume that all peers have the same downloading bandwidth.
$\theta \geqslant 0$ the rate at which downloaders abort the download.
$\gamma > 0$ the rate at which seeds leave the system.
η indicates the effectiveness of the file sharing, η takes values in $[0, 1]$. More details about η can be found in [9,14].

Here we assume that the seed departure rate $\gamma > 0$, which is always true in real networks since no peer will stay in the system forever. Note that in the unrealistic case $\gamma = 0$, seeds never leave the system. Hence eventually the number of seeds in the system will go to infinity and the system is unstable. In this paper, we always assume $\gamma > 0$.

The deterministic fluid model for the evolution of the number of peers (downloaders and seeds) used in [14] is given by

$$\frac{dx}{dt} = \lambda - \theta x(t) - \min\{cx(t), \mu(\eta x(t) + y(t))\},$$
$$\frac{dy}{dt} = \min\{cx(t), \mu(\eta x(t) + y(t))\} - \gamma y(t). \qquad (1)$$

The minimum operation in Eq. (1) determines whether the uploading bandwidth μ or the downloading bandwidth c is the constraint of the system. Because of the minimum operation, the whole system is a non-linear system.

Before we start the stability analysis, we will give some stability definitions [16] first. For a system $\dot{x} = f(x)$, where $f{:}R^n \to R^n$ may be linear or non-linear, a point x_e is an equilibrium point if $f(x_e) = 0$. The system is *globally stable* if for every trajectory $x(t)$, we have $x(t) \to x_e$ as $t \to \infty$. The system is *locally stable* near x_e if there is an $R > 0$, such that if the initial condition $x(0)$ is near the equilibrium point, i.e., $\|x(0) - x_e\| \leqslant R$, then $x(t) \to x_e$ as $t \to \infty$. Obviously, if a system is globally stable, then it must be also locally stable. The reverse is generally not true. However, if the system is linear, i.e., $f(x) = Ax$. then local stability is equivalent to global stability. A linear system is stable if and only if all the eigenvalues of the matrix A have negative real parts.

By letting $\frac{dx}{dt} = \frac{dy}{dt} = 0$ in Eq. (1), we can obtain the equilibrium point of the system as

$$\bar{x} = \frac{\lambda}{\beta\left(1 + \frac{\theta}{\beta}\right)},$$
$$\bar{y} = \frac{\lambda}{\gamma\left(1 + \frac{\theta}{\beta}\right)}, \qquad (2)$$