Journal of Hydro-environment Research 14 (2017) 19–33

Contents lists available at ScienceDirect

Journal of Hydro-environment Research

journal homepage: www.elsevier.com/locate/JHER

Research papers

# Numerical simulation of groundwater flow and land deformation due to groundwater pumping in cross-anisotropic layered aquifer system

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A R T I C L E  I N F O

*Article history:*
Received 26 January 2015
Revised 19 August 2016
Accepted 30 August 2016
Available online 21 September 2016

*Keywords:*
Hydraulic head
Land deformation
Pumping-recovery
Circular excavation
Heterogeneity

A B S T R A C T

In a natural aquifer, the aquifer properties are complicated, which normally vary with aquifer depth and geographic characteristics. Most previous studies have been limited to the case of isotropic aquifer with uniform characteristics. In the proposed model, the fully coupled ground water flow and land deformation due to groundwater pumping in cross-anisotropic aquifer system is used to analyze and predict. Based on this model, pumping-recovery tests in various conditions are numerically simulated to reveal the effects of cross-anisotropic aquifer behavior on hydraulic head and land deformation. Finally, the proposed method is applied to a project in Shanghai of China due to pumping of groundwater predicted. Comparing the calculated results with the measured field values indicates that the methods can be a useful tool for designing groundwater pumping projects.


## 1. Introduction

The technology of aquifer storage and recovery is widely applied for various purposes such as management and exploitation of fresh water resources, improvement of water quality, and excavation engineering to lower groundwater level. Inappropriate groundwater pumping, however, may cause earth fissures because artificial discharge of groundwater induces hydraulic forces that may impel the aquifer matrix to move vertically and horizontally.

To predict and prevent subsidence, various prediction methods on land subsidence had been proposed. Among these, Terzaghi (1925) solved the fully coupled groundwater flow and land deformation through conventional consolidation theory, in which solid skeleton deformation was forced to be uncoupled from groundwater flow by neglecting horizontal displacement. Biot's poroelastic consolidation theory (Biot, 1941, 1955) took the horizontal strain into consideration and solved the equations simultaneously. Then, the entire saturated-unsaturated flow regime by others (Noorishad et al., 1982; Bishop and Blight, 1963; Hsi and Small, 1992; Cai and Hu, 2010; Streltsova and Mckinley, 1984; Bear and Corapcioglu, 1981; Tarn and Lu, 1991; Yeh et al., 1996; Kim and Parizek, 1997, 1999, 2005; Lu and Jeng, 2007) had been further developed. Several previous researchers had discussed such problems in a single-layer aquifer system and layered aquifer system. Such as, Streltsova and Mckinley (1984) discussed the drawdown or buildup calculations for a well in a reservoir with various heterogeneous properties. Papadopulos and Cooper (1967) presented a closed-form solution for the drawdown in a large-diameter well which accounts for the effect of well storage. A set of type curves evaluated from their solution provided a useful tool to analyze pumping-test data for the aquifer parameters. Chang and Chen (2003) derived a solution for constant drawdown pumping test conducted on a well partially penetrating a confined aquifer of finite thickness. Perina and Lee (2006) derived a NUF solution for a well fully penetrating an unconfined or leaky aquifer without the discretization and with the radial flux across the well screen being a continuous function of depth. Bear and Corapcioglu (1981) developed a mathematical model for regional land subsidence due to groundwater pumping from a confined aquifer, and presented an analytical solution for the special case of a single well pumping from an infinite homogeneous isotropic confined aquifer, and they also did analogous work for a phreatic aquifer. Booker and Carter (1986, 1987) proposed an analytical solution for consolidation around a point sink embedded in a half-space with isotropic

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http://dx.doi.org/10.1016/j.jher.2016.08.001
1570-6443/

and anisotropic permeability, respectively. Another analytical solution was presented by Tarn and Lu (1991) for consolidation due to a point sink in a saturated half-space, which was cross-anisotropic with respect to both the hydraulic and mechanical properties. Yeh et al. (1996) constructed a Galerkin finite element model to simulate land subsidence resulting from groundwater pumping considering the unsaturated behavior and the anisotropic properties related to permeability and elasticity. Kim and Parizek (1997, 1999, 2005) simulated the rapid head rises at the beginning of pumping (i.e. the Noordbergum effect) and the rapid head drops at the start of recovery (i.e. the Rhade effect) in layered aquifer systems using a Galerkin finite element model considering both the variations in the saturated and unsaturated hydraulic properties during consolidation. Such abnormal groundwater level responses in the adjacent aquitards and unpumped aquifers at the beginning and the end of groundwater pumping had been referred to as reverse groundwater level fluctuations because they were opposite to the normal groundwater level responses in the pumped aquifers. Baú et al. (2004) compared numerical solutions of land subsidence due to fluid withdrawal from a semi-infinite medium whose boundary condition on the ground surface was respectively set to be permeable and impermeable. Kihm et al. (2007) simulated consolidation due to pumping from an unsaturated fluvial aquifer system with irregular lateral boundaries. Shen and Xu (2011) and Xu et al. (2012) predicted the future possible land subsidence in Shanghai by using the numerical analysis, in which the variation of the volume of groundwater, pumping layer, and pumping region through reallocation of pumping wells was considered. Budhu and Adiyaman (2013) developed the basic mechanics governing the changes in stress states from groundwater pumping and compared the predicted land subsidence from the mechanics developed with existing models and field data. The method of analysis could be used to predict land subsidence from variable groundwater level decline on a regional scale.

In a natural aquifer, the soil properties are complicated, which normally vary with aquifer depth, and geographic characteristics. Furthermore, anisotropic aquifer behavior is commonly observed in aquifer system. Therefore, it is necessary to investigate the effects of anisotropic and non homogeneous aquifer characteristics on fluid flow and solid skeleton deformation during pumping. On the other hand, previous results for dry elastic model and hydraulic properties apply, which constrain the properties derived for the coupled case where deformation and flow are linked. In this paper, considering the coupling of two kinds of deformation of solid skeleton and unsaturated water flow has the significant utility value.

The objective of this paper is to study hydraulic head changes and ground surface deformation due to pumping-recovery tests performed on a circular excavation in cross-anisotropic aquifer. Firstly, a mathematical model, which fully couples fluid flow and solid skeleton deformation in unsaturated porous elastic media, is formulated in axially symmetric cylindrical coordinates. The governing equations are solved simultaneously, which include both the variations in the saturated and unsaturated hydraulic parameters. Mechanical properties of diaphragm walls are also considered. The validity of the mathematical model is then verified by comparing its results with numerical solutions presented by Kim and Parizek (2005). Then, the influences of variable Young's modulus and cross-anisotropic parameters on hydraulic head and land deformation are studied. Finally, a case history of underground structure in Shanghai due to ground water pumping is carried out and compared with field data. These results can be useful for engineers to select serviceable supporting structures and reduce environmental impacts during the ground water pumping.

## 2. Mathematical model

### 2.1. Fully coupling the ground water flow equation and the equilibrium equations

A general mathematical model describing groundwater flow and land deformation due to groundwater pumping in unsaturated porous media can be formulated by fully coupling the flow equation and the equilibrium equations. The flow equation is expressed as (Kim and Parizek, 2005)

$$\nabla\cdot\left[-\boldsymbol{K}\cdot\nabla(H_{\mathrm{p}}+H_{\mathrm{e}})\right]+\left(n\frac{dS_{\mathrm{r}}}{dH_{\mathrm{p}}}+nS_{\mathrm{r}}\beta_{\mathrm{f}}\gamma_{\mathrm{f}}\right)\frac{\partial H_{\mathrm{p}}}{\partial t}$$
$$+\alpha_{\mathrm{c}}S_{\mathrm{r}}\frac{\partial}{\partial t}\left(\frac{\partial u_i}{\partial x_i}\right)=q_f,\quad i=r,z \tag{1}$$

where $H_{\mathrm{p}}=p_{\mathrm{f}}/\gamma_{\mathrm{f}}$ is the water pressure head, $p_{\mathrm{f}}$ is the pore-water pressure (position for compression), $\gamma_{\mathrm{f}}=\rho_{\mathrm{f}}g$ is the unit weight of the pore-water, $\rho_{\mathrm{f}}$ is the density of water; $H_{\mathrm{e}}=z$ is the head of water elevation; $n$ is the soil porosity; $S_{\mathrm{r}}$ is the degree of saturation; $\beta_{\mathrm{f}}$ is the compressibility of water; $\alpha_{\mathrm{c}}$ is Biot's coupling coefficient; $u_i$ is the soil displacement in the $r$-, and $z$-directions, respectively; $q_{\mathrm{f}}$ is the water sink term; $\boldsymbol{K}=k_{\mathrm{r}}k_{\mathrm{sat}ij}$ is the effective hydraulic conductivity tensor for $i,j=r,z$; $k_{\mathrm{r}}$ is the relative hydraulic conductivity; $k_{\mathrm{sat}ij}$ is the saturated hydraulic conductivity tensor. Generally, for $i\neq j, k_{\mathrm{sat}ij}=k_{\mathrm{sat}ji}=0$ and for $i=j$, $k_{\mathrm{sat}r}=k_{\mathrm{sat}\theta}=k_{\mathrm{sath}}, k_{\mathrm{sat}zz}=k_{\mathrm{satv}}$; $\mathrm{d}S_r/\mathrm{d}H_{\mathrm{p}}$ is the specific saturation capacity; and $nS_r$ is the volumetric water content.It is important to note that the Eq. (1) constitutes three dependent variables $H_{\mathrm{p}}$, $u_r$ and $u_z$. In Eq. (1), $\nabla\cdot(-\boldsymbol{K}\cdot\nabla H)$ can be expressed as

$$\nabla\cdot(-\boldsymbol{K}\cdot\nabla H)=-k_{\mathrm{r}}\left(k_{\mathrm{sath}}\frac{\partial^2 H}{\partial r^2}+k_{\mathrm{sath}}\frac{1}{r}\frac{\partial H}{\partial r}+k_{\mathrm{satv}}\frac{\partial^2 H}{\partial z^2}\right) \tag{2}$$

where, $H=H_{\mathrm{p}}+H_{\mathrm{e}}$ is the hydraulic head; $\nabla H$ is the hydraulic gradient; $\boldsymbol{v}_{\mathrm{r}}=-\boldsymbol{K}\cdot\nabla H$ is the Darcy velocity.

Substituting Eq. (2) into Eq. (1), Eq. (1) can be expressed as

$$-k_{\mathrm{r}}\left(k_{\mathrm{sath}}\frac{\partial^2 H}{\partial r^2}+k_{\mathrm{sath}}\frac{1}{r}\frac{\partial H}{\partial r}+k_{\mathrm{satv}}\frac{\partial^2 H}{\partial z^2}\right)$$
$$+\left(n\frac{dS_{\mathrm{r}}}{dH_{\mathrm{p}}}+nS_{\mathrm{r}}\beta_{\mathrm{f}}\gamma_{\mathrm{f}}\right)\frac{\partial H_{\mathrm{p}}}{\partial t}+\alpha_{\mathrm{c}}S_{\mathrm{r}}\frac{\partial\varepsilon_{\mathrm{v}}}{\partial t}$$
$$=q_f \tag{3}$$

in which

$$\varepsilon_{\mathrm{v}}=\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z} \tag{4}$$

where $\varepsilon_{\mathrm{v}}$ is the volumetric strain (positive for tension).

The equilibrium equations include two parts: the initial steady and the incremental stages. The initial steady stage can be omitted here since it is a stage of equilibrium initially. In axially symmetric cylindrical coordinates $(r,z)$, the equilibrium equations in the incremental stages can be rewritten as

$$\frac{\partial\sigma_r^1}{\partial r}+\frac{\partial\tau_{zr}^1}{\partial z}+\frac{\sigma_r^1-\sigma_\theta^1}{r}+f_r^1=0 \tag{5a}$$

and

$$\frac{\partial\sigma_z^1}{\partial z}+\frac{\partial\tau_{rz}^1}{\partial r}+\frac{\tau_{rz}^1}{r}+f_z^1=0 \tag{5b}$$

where superscript "1" denotes the incremental value of physical quantities; $f_r^1=0, f_z^1=[nS_{\mathrm{r}}\rho_{\mathrm{f}}+(1-n)\rho_{\mathrm{s}}]^1 g$ is the component of the incremental body force in the $r,z$ direction, respectively.