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The lateral distribution of depth-averaged velocity in a channel flow bend

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Abstract

This paper proposes an analytical model to predict the lateral distribution of streamwise velocity for flow in a curved channel with vertical sides, based on the depth-integrated Navier–Stokes equations. The model includes the effects of bed friction, lateral turbulence and secondary flows, where the additional secondary flow is approximated by a linear function of the lateral distance, as demonstrated by the limited data which are available. Two analytical solutions for the depth-averaged velocity are obtained, one for a flat bed and another for a bed with a transverse slope. Two parameters (denoted by *m* and *n* herein), which define the secondary flow, have been examined to analyze how they affect the velocity distribution in these two cases. Comparison of the analytical results with the limited experimental data available shows that the proposed model predicts the lateral distributions of depth-averaged velocity well. Further studies are needed to validate the values of the model parameters (*m* and *n*) for bends with different geometric properties.

Keywords: Velocity; Open channel flow; Curved channel; Channel bend; Hydraulics

1. Introduction

Since the early work of Rozovskii (1957), flow in channel bends has been studied by several researchers (e.g. Engelund, 1974; Ascanio and Kennedy, 1983; Yalin, 1992; Yeh and Kennedy, 1993; Jin and Steffler, 1993; Khan and Steffler, 1996). Noticeable development on the knowledge and understanding of the flow structure of bend are made recently (e.g. Blanckaert and Graf, 2001, 2004; Blanckaert and de Vriend, 2003, 2005, 2010; Blanckaert et al., 2008; Constantinescu et al., 2011, 2013; Jamieson et al., 2010, 2013; Kashyap et al., 2012; Sukhodolov, 2012; Ottevanger et al., 2013). The experimental data of velocity in a bend show two circulation cells in a cross section: alongside the classical helical motion (centre-region cell), a weaker counter-rotating cell (outer-bank cell) is formed in the corner of the outer bank near the water surface (Blanckaert and Graf, 2001). Blanckaert and Graf (2004) found the advective momentum transport by the central-region cell significantly affects the velocity profile and bed shear stress in a sharp bend, by evaluating each term of the momentum equations using available experimental data. The cross-stream circulation in a bend was further studied by detailed 3D velocity measurements (Blanckaert et al., 2008; Jamieson et al., 2010, 2013). van Balen et al. (2009) and Stoesser et al. (2010) analyzed the pattern of secondary flow cell in a bend and its impact on the bed shear stress in terms of 3D numerical modelling, while others have developed simpler, 2D models, e.g., Hsieh and Yang (2003). With respect to 2D modelling, Blanckaert (2005) stated that "... conventional depth-averaged two dimensional (2D) models are intrinsically unable to account for the secondary flow ...". Blanckaert's statement has highlighted the need for a secondary flow correction when 2D models are applied to simulate flow in bends, particularly by supplementing the 2D models with a closure sub-model for the secondary flow.

Camporeale et al. (2007) have given a review on commonly used simple models for curved rivers to illustrate the interconnected processes among the hydrodynamics, bed morphodynamics and bank morphodynamics. Most existing

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hydrodynamics models for curved channel flow take account for the impact of secondary flow using a parameterization that is based on the hypothesis of mild curvature. For example, Johannesson and Parker (1989) used a perturbation expansion to linearize the depth-integration momentum equation, where the secondary flow was parameterized with an empirical shape function for the vertical distribution of primary velocity. Thus based on the linearity and gradual variation assumptions, the representation of the secondary flow is justified for small curvature channels, but it is not appropriate in moderately and strongly curved bends. For strongly curved bends, Blanckaert and de Vriend (2003, 2010) proposed a sub-model for a non-linear treatment of the secondary flow in the depth-averaged momentum equation, where the secondary flow was parameterized through a correction factor which depends on the so-called bend parameter. Thus the derived nonlinear sub-model was a reduced-order equation. Ottevanger et al. (2013) extended the non-linear model of Blanckaert and de Vriend (2010) to the bed morphology in strongly curved bends. However, it is worth noting that all these sub-models were not directly to resolve the 2D depth-averaged momentum equations rather than they further reduced the equations by taking the average or the first moment in lateral direction.

Similar arguments have often drawn attention against quasi 2D models in straight channels, e.g., the Shiono & Knight model (SKM) (Shiono and Knight, 1991; Tang and Knight, 2008a,b, 2009) has often been accused of being too simplistic, yet it has yielded significant insight into the flow dynamics in straight channels and has enabled reasonably accurate predictions of depth-averaged velocities, boundary shear and channel discharge to be made in natural rivers (Abril and Knight, 2004; Knight et al., 2007, 2010a,b; Knight, 2012; Sharifi and Sterling, 2009). The aim of this paper is to illustrate that it is possible to use the SKM to accurately predict the lateral distribution of depth-averaged velocities in bends by solving the depth-averaged momentum equation.

Before describing the mathematics relating to the SKM model it will be beneficial to outline in qualitative terms the secondary flow associated with a number of features inherent in bend flow. Since this discussion is purely qualitative, flow around a "general" bend will be discussed, as illustrated in Fig. 1. It is reasonably straightforward to demonstrate that a lateral pressure gradient exists between the inner and outer banks which are proportional to the local streamwise velocity squared and inversely proportional to the distance from the centre of curvature of the channel. Combined with the effect of boundary layer drag arising from the channel bed, the flow in a curved channel will have some relatively large scale vorticity, resulting in what is commonly termed 'Prandtl's secondary flow of the first kind' (Schlichting, 1979). Fig. 1 illustrates that the flow can be dominated by one large scale secondary flow cell which extends from the inner bank to cover most of the cross section and a corresponding small secondary flow cell near the water surface at the outer bank. This is remarkably different from what is known to exist in a straight channel where a series of secondary flow cells arise as a result of the anisotropy in Reynolds stresses (Nezu et al., 1993; Albayrak and Lemmin, 2011). Fig. 2 illustrates that at a particular cross section the flow can be conceptualized to consist of a number of different secondary flow cells which ensure that lateral gradient of $(UV)_d$ (where U represents the local streamwise velocity, V represents the local transverse velocity and the subscript d denotes a depth average) varies across the channel. A full explanation of how $(UV)_d$ is associated to the secondary cell can be seen in the paper by Knight et al. (2007). Subsequent sections will discuss the importance of $(UV)_d$ but for now it is sufficient to note that it can be interpreted as an indicator of the strength of secondary flow cell. Hypothesizing this distribution for the case illustrated in Fig. 1 would suggest that the flow domain could be discretized into four panels – two large panels corresponding to the large secondary flow cell and smaller panels centred around the outer bend secondary flow cell. Indeed, given the relative size of the both flow cells it could be postulated that it is not unreasonably too simple to just use a two panel structure, i.e., effectively ignoring the contribution of the smaller flow cell. The feasibility of this is addressed below.

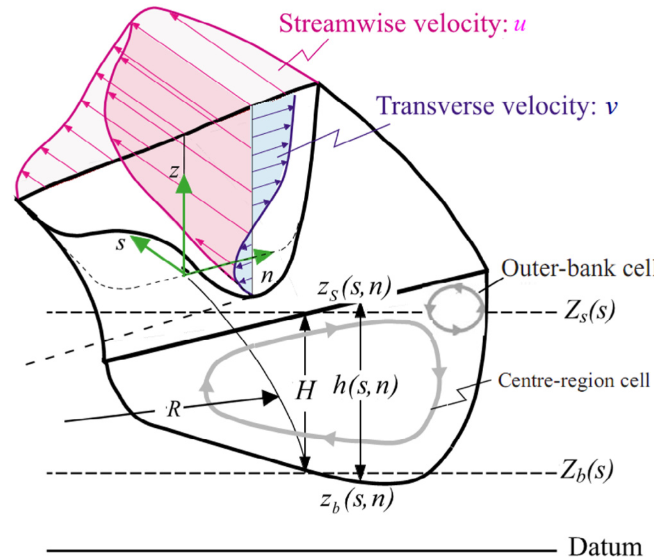

Fig. 1. A typical flow structure in a bend.Modified after Blanckaert and de Vriend, 2010.

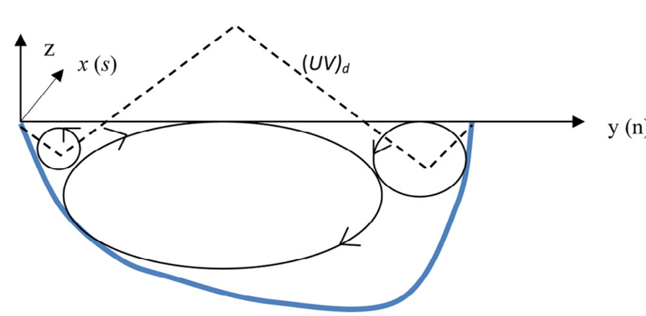

Fig. 2. A sketch of a cross-section in a bend with secondary flow cells, where a flat surface is assumed as simplicity.