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Research paper

Numerical and experimental turbulence studies on shallow open channel flows

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Abstract

The standard shallow water equations (SWEs) model has been proven to be insufficient to consider the flow turbulence due to its simplified Reynolds-averaged form. In this study, the k-ε model was used to improve the ability of the SWEs model to capture the flow turbulence. In terms of the numerical source terms modelling, the combined SWEs k-ε model was improved by a recently proposed surface gradient upwind method (SGUM) to facilitate the extra turbulent kinetic energy (TKE) source terms in the simulation. The laboratory experiments on both the smooth and rough bed flows were also conducted under the uniform and non-uniform flow conditions for the validation of the proposed numerical model. The numerical simulations were compared with the measured data in the flow velocity, TKE and power spectrum. In the power spectrum comparisons, a well-studied Kolmogorov's rule was also employed to complement both the numerical and experimental results and to demonstrate that the energy cascade trend was well-held in the investigated flows.

Keywords: k-ε model; Laboratory experiment; Power spectrum; Rough bed flow; Smooth bed flow; SGUM model; SWEs model; TKE

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1. Introduction

The shallow water flows are of significant theoretical and practical interests and various experimental and numerical works have been carried out to address different issues involved, such as in Cheng et al. (2012), Liang et al. (2007) and Lin et al. (2011). To compensate the simplicity of shallow water numerical model to reproduce the flow turbulence, various turbulence equations have been investigated and one of the most common applications of such approach is in the dam-break flows as presented in Ozmen-Cagatay and Kocaman (2010). The two common turbulence shallow water equations (SWEs) were usually based on the principles of 1) variation of the turbulent eddy viscosity, such as the turbulent energy and energy dissipation related models (Rodi, 1993), and 2) variation of the Reynolds stress, such as the Reynolds stress models (RSM) (Launder et al., 1975). There are also some other models, which were derived from both the k-ε and RSM equations, namely the Launder and Ying (LY) model (1973). The model was first suggested by Launder and Ying (1973), and further tested in Shiono et al. (2003).

The flow turbulence equations that involve the Reynolds stress modelling, such as the RSM and LY equations, are more suitable to represent the highly turbulent flows, e.g. in the vegetated flows of Choi and Kang (2004) and compound channel flows of Shiono et al. (2003), where the secondary currents are present. However, from the formulation of the turbulence equations, it can be observed that the RSM equations are more computationally expensive than the k-ε equations (refer to the full explanations of RSM concept in Shiono et al., 2003; Choi and Kang, 2004). As for the wide

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channel flows with less intense eddy formations, e.g. the shallow water flows, the k-ε equations have been proven to well-represent the flow turbulence. Studies of shallow water k-ε models under different flow conditions can be found in Gomez (2005) and Erpicum et al. (2009). In more specific applications, Nassiri (1999) and Pu et al. (2012a) have applied the shallow water k-ε model to represent the recirculating and obstruction-induced flows, respectively, and good computational results were obtained in their studies when the flow turbulence was less intensive. The comprehensive studies of various shallow water k-ε models were also conducted in Rodi (1993) and more recently in Gomez (2005).

On the other hand, the use of power spectrum to indicate the mechanism of turbulence and turbulent kinetic energy (TKE) cascade system was first expressed in a universal manner by Kolmogorov (1941a, 1941b, 1941c). In his works, the power spectrum was categorised into three ranges: the non-viscous subrange, inertial subrange, and viscous range. The non-viscous subrange is a short range, which in a relatively small experimental flume with small width, e.g. studied by Kironoto and Graf (1995), it only lasts for one decade from 0.1 Hz to almost 1 Hz. Under this range, all the energy is supplied by the external sources/forces, hence an almost constant energy cascade pattern can be observed.

In the inertial subrange, the energy cascade occurs in the locally isotropic condition (Kolmogorov, 1941a). Deriving from the 2/3 law suggested by Kolmogorov (1941a) for the second moment of turbulence, he further proposed a universal slope of −5/3 for the power spectrum in this subrange for the large Reynolds number (Re) flow (Kolmogorov, 1941b, 1941c; Batchelor and Townsend, 1949). This law of −5/3 slope, also called the K41 scaling rule, has later been investigated by many researchers and proved for its universality in different flow conditions (Nezu and Nakagawa, 1993; Frisch, 1995; Mordant et al., 2004; Blanckaert and Lemmin, 2006).

When the flow Reynolds number is large, the flow energy-contained eddies are larger than the dissipating eddies, and thus it causes a broader inertial subrange (Frisch, 1995). Since the inertial subrange is located between the non-viscous and viscous ranges, its energy dissipation trend has a significant influence on the whole flow energy cascade system. In Kironoto and Graf (1995), the inertial subrange was found to start at about 1 Hz and end at about 10 Hz for the flows in their relatively small flume with 0.6 m width.

In the outmost viscous range, the TKE dissipation causes a fast energy cascade and the slope value of its power spectrum is dependent on each specific flow condition. Usually it should be greater than that in the inertial subrange due to its high turbulent viscosity (Tchen, 1953; Frisch, 1995). More recently, the energy cascade system and the Kolmogorov's rule have been investigated and proven numerically by Aristov and Rovenskaya (2011) using a Boltzmann kinetic turbulence model; while, the mechanism of large and small scale eddies and their effects on the energy cascade system were studied theoretically by Hunt et al. (2010).

Conclusively, the whole three-range power spectrum describes the energy cascade process starting from the flow's energy receiving (associated with its large eddies in low-frequency), and then evolving to the flow's energy dissipating (associated with its reformation into the small eddies with high-frequency). Out of these three turbulence ranges, the inertial subrange has the most universal energy cascade pattern, hence in the present study, the focus was put on to investigate this subrange.

A lot of useful works have been done in the standard turbulence k-ε modelling with the SWEs, so in the current work we proposed a numerical SWEs k-ε modelling improvement by using a documented source terms treatment method into the standard 2D finite volume scheme. This improvement was accomplished by integrating a 1D surface gradient upwind method (SGUM) as proposed by Pu et al. (2012b) into the present 2D SWEs k-ε model, as the SGUM is robust to improve the efficiency of numerical scheme that was burdened by the additional TKE source terms. The use of SGUM can improve the extra turbulence source terms simulation in the SWEs k-ε model by integrating them into the main upwind schemes commonly used to update the numerical flux terms (Pu et al., 2012b). An experimental study under different flow conditions has also been carried out to validate the proposed modelling results. Both the numerical and experimental models were used to investigate the shallow water flows under smooth and rough bed conditions. The numerical simulations were compared with the experimental measurements in terms of the flow velocity, TKE and power spectrum. Besides, the SWEs computed turbulence power spectrums were also compared with the Kolmogorov's (1941a, 1941b, 1941c) K41 scaling rule in the inertial subrange, for which very limited works have been documented in the SWEs literature.

2. Numerical models

2.1. Shallow water equations model

In this study, the SWEs model is used to couple with the turbulence k-ε model. The 2D fully conservative shallow water equations are presented in equations (1)–(3), and it is combined with the numerical flux terms from the 2D k-ε model.

$$\frac{\partial \phi}{\partial t}+\frac{\partial \phi u}{\partial x}+\frac{\partial \phi v}{\partial y}=0 \tag{1}$$

$$\begin{aligned}\frac{\partial \phi u}{\partial t}+\frac{\partial\left(\phi u^2+\phi^2/2\right)}{\partial x}+\frac{\partial \phi uv}{\partial y}-\frac{\partial}{\partial x}\left[2\nu_t\frac{\partial(\phi u)}{\partial x}-\frac{2}{3}\phi k\right]\\ -\frac{\partial}{\partial y}\left[\nu_t\left(\frac{\partial(\phi u)}{\partial y}+\frac{\partial(\phi v)}{\partial x}\right)\right]=g\phi\left(S_{ox}-S_{fx}\right)\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{\partial \phi v}{\partial t}+\frac{\partial \phi uv}{\partial x}+\frac{\partial\left(\phi v^2+\phi^2/2\right)}{\partial y}-\frac{\partial}{\partial x}\left[\nu_t\left(\frac{\partial(\phi u)}{\partial y}+\frac{\partial(\phi v)}{\partial x}\right)\right]\\ -\frac{\partial}{\partial y}\left[2\nu_t\frac{\partial(\phi v)}{\partial y}-\frac{2}{3}\phi k\right]=g\phi\left(S_{oy}-S_{fy}\right)\end{aligned} \tag{3}$$