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Numerical modeling of wave propagation into a harbor using improved characteristic scheme

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Abstract

The method of characteristics has been widely used for solving free surface flows in $X-T$ and $X-Y-T$ spaces. Applications of the method to simulate tidal waves, landslide generated waves, and tsunamis indicate that the method is reliable for solving long wave propagation and its related problems. Many other researches indicated that the method is also capable of simulating more challenging problems of relatively short wave propagation into a harbor. Lately an interpolation scheme was developed and tested for simulating wave oscillation and propagation. The technique was shown to be better than the previous characteristic schemes. In this paper, the method is applied for simulating wave propagation in a harbor where the numerical scheme has to deal with wave reflection, refraction and diffraction. Maximum wave heights inside and outside the harbor basin were recorded. Simulation of wave propagation into the same harbor (basin) using commercial software (Surface Water Modeling System, CG Wave Module) is used for comparison. The comparison indicates that the improved characteristic scheme is a good tool for simulating wave propagation into harbor basins.

Keywords: Simulation; Shallow water; Long wave; Numerical dissipation; Interpolation

1. Introduction

Due to the increasing demand of harbors for sea transportation, and fishing the government as well as private companies in Indonesia have to develop new harbors or expand their existing harbors. Numerical or physical simulations are necessary to assure that the designs meet the requirement of wave height limitation within the harbors. In addition, the simulations may also be advantageous to predict and mitigate the effect of harbor developments on coastal environment. Problems of wave penetrations into a harbor basin have been successfully dealt with using both physical and numerical approaches. The physical experiments, although much more expensive than the numerical approach are still in use in many situations especially when complicated wave dynamic is expected in the model. For example Kofoed-Hansen et al. (2000) utilized both numerical and physical models to study seiching in exposed new marina where the physical model was applied to investigate especially for incident wave conditions associated with strong wave breaking, where the numerical model was not yet feasible. He et al. (2006) simulated the performance of a combined sewer overflow using physical and numerical models. The physical model was used to establish a verified numerical model of the facility for future work.

Despite the limited capacity to represent all the parameters and complexity of the physical conditions, the numerical approach has won the competition in certain applications between the two approaches and gain popularity since 1980s. Since then and with the advent of faster personal computer, numerical simulation approach has been increasingly advanced. A number of software based on finite element or finite difference approaches are made available in the market making the numerical simulation even more accessible to engineers.

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The method of characteristic is among others, a very good tool to solve hyperbolic equations. The numerical version however, requires fixed Cartesian grid size to avoid complexity of the solution. In $X-T$ space, the method was introduced in 1953 (Hartree 1953). The characteristics scheme in $X-Y-T$ has been developed and found many applications in tidal waves simulations since 1967. However it is highly dissipative when applied to a computational domain where the number of the grids is relatively small compare to the wavelength. Such dissipative nature may be one of the reasons why the method is not as popular as other methods of computation.

Improvement of the characteristics scheme based on quadratic interpolation has made the method significantly less dissipative (Triatmadja 1990, 2008). Simulation of wind wave in harbors and coastal areas using the method seems to be feasible. Hence, it is important to study the performance of the scheme for simulating shallow water wave problems such as wave penetration and propagation into a harbor where relatively large numbers of grids are required in the computational domain.

2. The development of characteristics scheme in $X-Y-T$ space and its performance

The method of characteristics in $X-Y-T$ space was originally developed by Butler (1960) whilst its numerical scheme in $X-Y-T$ space was first introduced by Townson (1967).

The number of characteristics drawn back from an arbitrary point in $X-Y-T$ space is infinite which form a conoid. The depth averaged equation, (Eq. (1) through Eq. (3)) that govern the shallow water dynamic may be solved using selected four characteristics emanate from the crest of the characteristics conoid. In a fixed grid numerical scheme the crest of the characteristic scheme is for example a grid point (i,j) at time plane $t+\Delta t$ (Fig. 1)

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}+g\frac{\partial \eta}{\partial x}+\frac{c_f}{h}U\sqrt{U^2+V^2}=0 \tag{1}$$

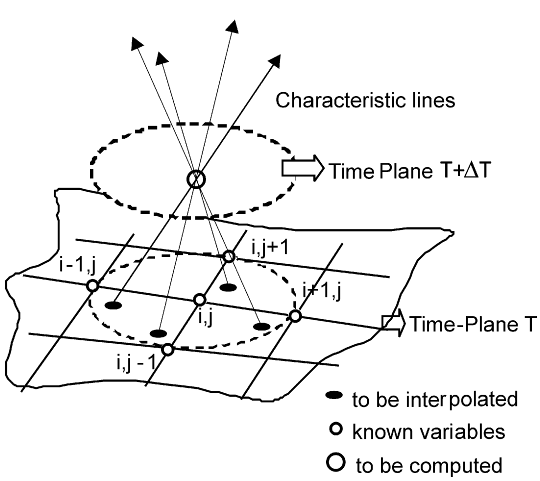

Fig. 1. Four selected characteristics penetrate time-plane t.

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}+\frac{\partial \eta}{g\partial y}+\frac{c_f}{h}V\sqrt{U^2+V^2}=0 \tag{2}$$

$$\frac{\partial h}{\partial t}+\frac{\partial hU}{\partial x}+\frac{\partial hV}{\partial y}=0 \tag{3}$$

where $h=$ the water depth; $\eta=$ water surface fluctuation relative to still water level; $g=$ gravitational acceleration; U and $V=$ depth average velocity in X and Y directions respectively while the last terms of the left hand side of Eq. (1) and Eq. (2) are frictional components.

The method of characteristics has certain advantages over other numerical techniques yet suffers from significant drawback due to the need of interpolation when applied to fixed grid system. The dissipative nature of the characteristics scheme is partly due to the inaccurate interpolation. In order to solve the characteristics equations in $X-Y-T$ space one should consider four characteristics along which the differential equations are valid. These four characteristics that represent all other characteristics when drawn backward from time $t=t+\Delta t$, do not always penetrate the time-plane t exactly at the grid points where the variables are known (Fig. 1). Hence, interpolation is required. The interpolation may employ grid points $(i-1,j)$, $(i+1,j)$, $(i,j-1)$ and $(i,j+1)$ of Fig. 1.

Such approximation has made the method dissipative that reduces its attractiveness especially at low resolution where the waves are represented by relatively small number of grids. The linear interpolation scheme is as follows.

$$U_{i,j}^{t+\Delta t}=\overline{U}-g\eta_x\Delta t-\left(U_{i,j}V_x+U_{i,j}U_y\right)\Delta t-gU_{i,j}S_{i,j}\Delta t \tag{4}$$

$$V_{i,j}^{t+\Delta t}=\overline{V}-g\eta_y\Delta t-\left(U_{i,j}V_x+V_{i,j}V_y\right)\Delta t-gV_{i,j}S_{i,j}\Delta t \tag{5}$$

$$\eta_{i,j}^{t+\Delta t}=\overline{\eta}-\left(U_{i,j}h_x+h_{i,j}U_x+V_{i,j}h_y+h_{i,j}V_y\right)\Delta t \tag{6}$$

where $U_{i,j}$ and $V_{i,j}=$ velocities at grid point i,j in X and Y directions respectively; $\bar{U}$, $\bar{V}$, or $\bar{\eta}=$ average value of the variables at $(i-1,j)$, $(i+1,j)$, $(i,j-1)$ and $(i,j+1)$; $U_x,U_y,V_x,V_y,\eta_x,\eta_y$ may be represented by Q_x and Q_y as follows.

$$Q_x=\left(Q_{i+1,j}-Q_{i-1,j}\right)/2\Delta s;\ Q_y=\left(Q_{i,j+1}-Q_{i,j-1}\right)/2\Delta s;$$

$\Delta s=\Delta x=\Delta y=$ grids size in X or Y direction; $\Delta t=$ time step; $S_{i,j}=\frac{\sqrt{U^2+V^2}}{C_z^2h}$; $Cz=$ Chezy coefficient.

The scheme is explicit and stable when the Courant number (Cr) in Eq. (7) less than unity.

$$Cr=C\frac{\Delta t}{\Delta s}\sqrt{2} \tag{7}$$

where $C=$ wave celerity ($C=\sqrt{gh}$).

The amplitude portrait of the original scheme where $a_1=$ wave amplitude after one oscillation; $a_0=$ original wave amplitude and $L=$ wave length, is given in Fig. 2.

Clearly, Fig. 2 indicates that the numerical wave is very dissipative even for $L/\Delta s$ as many as 100 and as long as the Cr is less than 1.0. However the technique is satisfactory when simulating tidal waves where the size of the computational