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Stochastic comparisons: A methodology for the performance evaluation of fixed and mobile networks

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ABSTRACT

We propose to use a mathematical method based on stochastic comparisons of Markov chains in order to derive performance indices bounds. The main objective is to find Markovian bounding models with reduced state spaces, which are easier to solve. We apply the methodology to performance evaluation of complex telecommunication systems modelled by large size Markov chains which cannot be solved by exact methods. This methodology can be applied for continuous- or discrete-time Markov chains. In the first study, we consider an MPLS switch represented by two stages of buffers. Various kinds of traffic with different QoS levels enter the first stage, and transit in the second stage. The goal is to compute packet loss rates in the second stage. In the other study, we define a CAC scheme in a mobile network which gives the priority to the handover over the new calls. Performance evaluation of the CAC scheme consists in the computation of the dropping handover and call blocking probabilities. For the two studies, systems are represented by large state Markov chains whose resolution is difficult. We propose to define intuitively bounding systems in order to compute performance measures bounds. Using stochastic comparisons methods, we prove that the new systems represent bounds for the exact ones. Different methods can be used. For the MPLS switch, we use the coupling equivalent to the sample-path ordering, allowing the comparison of the loss rates. In the case of the CAC scheme, we apply the increasing sets formalism used to define weaker orderings, enabling the comparison of the dropping handovers and blocking probabilities. We validate stochastic comparison method by presenting some numerical results illustrating the interest of the approach.

1. Introduction

With the variety of information traffic flows and the growing complexity of telecommunication systems, it will be very difficult to guarantee a sufficient QoS level to all users. Performance analysis of these systems is crucial but also very hard to carry out.

Since quantitative analysis is based on multi-dimensional processes, the study by classical methods such as simulation or mathematical analysis, if there is no specific solution, can be difficult due to the state space explosion. As exact performance measures can only be obtained using numerical methods [17] with small sizes, thus it is important to develop powerful new mathematical tools to analyze large size Markov processes.

We propose to use a mathematical method based on stochastic comparisons of Markov chains. The key idea of this method is that given a complex Markov chain, which cannot be analyzed numerically, we propose to bound it by a new Markov chain which is easier to solve.

Given a performance measure to be computed, the stochastic comparison method is composed by the following main steps:

- the choice of the state space of the comparison;
- the choice of the relation order on the chosen space;
- construction of bounding models which are easier to solve by simplifying the original system: for example changing input parameters, or reducing the number of components of the state, etc. It is clear that the quality of the bounds depends on the choice of the bounding models. It is very difficult to be sure in the step of the definition of the bounds that the bounding system is a "good" bound, this can be verified only in the test phase with several numerical results.

As we will see in this paper, stochastic comparisons represent an interesting mathematical tool for the performance evaluation of large size Markov chains. We use this methodology in order to define bounding Markov chains on a smaller state space, in order to compute bounds on performance measures. The advantage of

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this method is that it can be applied for many kinds of networks. We have already obtained some interesting results for mobile networks, MPLS/IP network, and optical networks. In [3,1,2], we applied this method on mobile networks in order to obtain dropping handover bounds. In [5], we used it to compute packet loss rates in an optical switch, and in [4] for the packet loss rates in an IP switch. [13] presents this method in details and applies it to evaluate cell loss rates in an ATM switch.

To illustrate the usefulness of this method, we present applications of this method in the case of fixed networks and mobile networks.

In the first application, we study an MPLS switch. The objective is to compute the packet loss rates in the buffers. It is an important performance measure in broadband integrated services digital networks, as it may be a part of the contract on the quality of service (QoS) between the user and the network provider. In this case, we need to solve a multi-dimensional discrete-time Markov chain which is very difficult because of the state space explosion problem. We propose to build two bounding models on smaller state spaces providing a lower and a higher bound of the packet loss rate. We use the coupling method applied to discrete-time Markov chains for the stochastic comparison of the bounding models with the exact system. The coupling method of Markov chains is equivalent to a sample-path ordering. It is quite easy to carry out as it remains to compare the states of the processes at each instant, for each event happening, according to the order defined on the state space. The coupling method generates the strong stochastic ordering denoted $\preceq_{st}$ which allows us to compare the loss rates written as increasing reward functions on the stationary distribution.

In the second application, we study a CAC (Connection Admission Control) scheme for mobile networks. This scheme takes into account different types of traffic (e.g. voice and data). To model this scheme, we built a multi-dimensional continuous-time Markov chain which is very difficult to solve because of the state space explosion problem. We built a smaller Markov chain in order to compute dropping handover and call blocking probabilities bounds. The stochastic comparison is based on the increasing sets method for continuous-time Markov chains. It is more difficult to carry out than the coupling because we have to define all the increasing sets belonging to a family, but it allows us to define weak orderings which are less constrained then the strong ordering. The weak ordering $\preceq_{wk}$, allows the comparison of the tail distribution functions and also performance measures as dropping handover and call blocking probabilities.

This paper is organized as follows: in next section, we give an outline of the stochastic comparison methodology in a performance study. We also explain the two main stochastic comparison methods used in this paper: the coupling and the increasing sets. In Section 3, we present the MPLS switch model and we give the model of CAC scheme in mobile networks. In Section 4, we explain how to define bounding models providing bounds on packet loss rates in MPLS switch, and on dropping handover and on blocking probabilities in CAC scheme. Numerical results for both applications are given in Section 5. Finally, we present the main contribution of this work and give future prospects.

2. Stochastic comparison methodology

Stochastic comparison[11,14,9,6] is a mathematical tool used in the performance study of systems modelled by continuous- or discrete-time Markov chains. The general idea of this method is to bound a complex system by a new system, easier to solve and providing performance measures bounds.

2.1. Performance measure bounds

In a lot of cases, we have to study multi-dimensional processes, whose evolution is not always easy to know due to the number of components, and events which happen. If we are able to define the state evolution, another problem to solve is the computation of the stationary distribution.

If there is not a product form, the stationary distribution is not easy to compute if the number of states is high.

Stochastic comparison proposes to solve this problem by reducing the size of the Markov chain. The goal of the performance study is to compute performance indices. We focus on performance indices computed as increasing reward functions (according to the order defined on the state space of the Markov chain) on the stationary distribution. In a lot of cases, this reward function is the same for a set of states (so it does not need all the components), or it depends only on a few states.

From this remark, we can say that it is not necessary to represent all the states, and some of them can be put together (for example which have the same value of the reward function), in order to reduce the size of the state space. The stochastic comparison proposes to define a new Markov chain which is an aggregation of the initial one. Aggregation process consists in the state space reduction, in other words by making together some states and to map them into only one state. Clearly, state space reduction can be performed intuitively by simplifying the original system (reduction of the component number, modifying some properties, etc.) or by applying an automatic aggregation algorithm [18]. In this paper, we define bounding systems intuitively, by considering the performance measure to be computed, and the parameters which can influence them. Next, we explain formally the state space reduction.

Let I be a performance measure computed as an increasing reward function f on the stationary distribution of a continuous-time (or a discrete-time) Markov chain $X(t)$ defined on a multi-dimensional state space E:

$$I = \sum_{x \in E} \Pi(x)\rho(x)$$

We suppose that Π is very difficult to compute (there is no product form, and the number of states is high).

We denote by $\preceq$ the order defined on E, we suppose for example that it is the partial order component by component. The choice of the order is very important because the stochastic comparison theory imposes that the function ρ $(E \rightarrow R_{>0})$ must be increasing according to it.

We suppose that $\rho(x)$ do not depend on all states of E which means that it depends only on some components of x. We denote by S the set of these states, containing enough information for the computation of R. We propose to define bounding processes intuitively on a smaller set then E in order to solve them easier.

2.2. Stochastic comparison by mapping functions

As $X(t)$ is very difficult to solve, and R depends only on some components, we propose to define smaller bounding processes which are easier to solve, from which we can compute bounds for R, using the same reward function $\rho(x)$. State spaces of bounding processes are also multi-dimensional, but with less components, we propose to use the same preorder defined on E. Two solutions are proposed, the simpler solution consists in finding a bounding process directly on S. This solution is simple, but the bounding process could be too far from the exact process, especially if S is a very reduced state space. This solution is the first one presented just after. The second solution is more general and can give more precise bounds. The bounding process is derived also