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An improved broadband method for the evaluation of effective parameters of slab metamaterials

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Abstract

An improved broadband method for determining complex effective refractive index, dielectric and magnetic constants of an arbitrary passive metamaterial has been proposed. Evaluation of the effective parameters is realized using the reflection–transmission S-parameters obtained by simulation or experimental measurements and analytically evaluated interface reflection coefficient of the slab.

Comparison of precision of the presented method with the Nicolson–Ross techniques [Nicolson AM, Ross GF. Measurement of the intrinsic properties of materials by time-domain techniques. IEEE Trans Instrum Meas 1970;TM-19(4):377–82; Ghodgaonkar DK, Varadan VV, Varadan VK. Free-space measurement of complex permittivity and complex permeability of magnetic materials at microwave frequencies. IEEE Trans Instrum Meas 1990;39(2):387–94] has been made using the simulations for different configurations of rod metamaterials. Some discussion concerning the sensitivity of the effective parameters of metamaterials for the accuracy of the frequency dependent S-parameters is also presented in this paper.

Keywords: Metamaterials; Effective parameters; Effective medium theory; Microwave; Finite difference time domain

1. Introduction

Retrieval techniques are nondestructive and contactless methods, to evaluate the effective dispersive parameters of an inhomogeneous material. They have a big practical interest due to fast development of metamaterials in the last decade. Majority of these techniques are modified retrieval of so-called Nicolson–Ross method [1,2], very well developed for parameter characterizations of materials on microwave frequencies. Development of such techniques is especially important for microwave frequency range because the desirable properties of conventional materials are seriously degraded for frequency above 1 GHz [3]. Such techniques are, as usual, based on effective medium theory approach [4,5]. Knowledge of microwave dispersive parameters of metamaterials is very important in order to design a variety of microwave circuits such as circulators, phase shifters and filters.

Retrieval parameter techniques give a possibility to evaluate effective dielectric constant ε_{eff} and magnetic constant μ_{eff} of metamaterials implying that effective refractive index should be evaluated as $\sqrt{\varepsilon_{\text{eff}}\mu_{\text{eff}}}$, but the techniques do not mention how to define correct branch of square root with mathematical viewpoint. Moreover, the calculation procedure of techniques requires choosing the complex function branch of square root inclusive of function of S-parameters like

$$\Gamma(S_{11}, S_{21}) = K(S_{11}, S_{21}) \pm \sqrt{K^2(S_{11}, S_{21}) - 1}.$$

The procedure choice implies that $|\Gamma(S_{11}, S_{21}) < 1|$. Sometimes this way to choose the complex branch gives mistaken

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result at some test frequencies since both possible branches satisfy the above conditions. In this paper, we propose a method that does not require making a choice of square root branch of complex functions of evaluated S-parameters in order to evaluate any of the refractive index, permittivity or permeability. Instead, we obtain a formula for evaluation of complex refractive index which is simple.

It is well known that Nicolson–Ross like techniques have been designed for macroscopically homogeneous materials but an evaluation of effective parameters of metamaterials sometimes requires an introduction to the boundaries of effective flat material. This problem is being discussed in this paper.

Assessment and precision of the presented method is being made using a comparison of the simulation results obtained by our method and the Nicolson–Ross one. Some discussion concerning the sensitivity and precision of the evaluating effective parameters of metamaterials for the accuracy of the frequency dependent S-parameters is also presented in the paper.

2. Evaluation method

Consider normal incidence of an electromagnetic wave on a slab. We consider an inhomogeneous slab of thickness which is inhomogeneous in the direction of the incident wave. We can, in principle, replace this inhomogeneous slab with a homogeneous slab of the same thickness. This approach is called "the thin sample principle", i.e. we imply that the thickness of the metamaterial sample is much less than the other dimensions of the same. Regardless of the fact that the field produced by the induced currents is not uniform, we can consider a field beyond the slab as a plane wave like field for the incident wave. This approach is quite acceptable for frequencies up to 10 GHz [1,2,6–10] and is in the domain of the effective medium theory valid for the case of a slab like inhomogeneous samples. S-parameters of an inhomogeneous slab can be described closely equal to [1,11]

$$S_{11} = \frac{(1 - \mathrm{e}^{-2\mathrm{i}k_0 n_{\mathrm{eff}}(\omega)d})R(\omega)}{(1 - R^2(\omega)\mathrm{e}^{-2\mathrm{i}k_0 n_{\mathrm{eff}}(\omega)d})},$$

$$S_{21} = \frac{(1 - R^2(\omega))\mathrm{e}^{-\mathrm{i}k_0 n_{\mathrm{eff}}(\omega)d}}{(1 - R^2(\omega)\mathrm{e}^{-2\mathrm{i}k_0 n_{\mathrm{eff}}(\omega)d})}, \qquad (1)$$

where ω is a frequency of the incident electromagnetic wave, $R(\omega)$ is the interface reflection coefficient of the slab, k_0 is the wavenumber of the incident wave outside the slab and n_{eff} is effective refractive index of the slab under investigation.

Leaving out the term $\mathrm{e}^{-2\mathrm{i}k_0 n_{\mathrm{eff}}(\omega)d}$, expression (1) can be rewritten as

$$-\mathrm{i}\frac{\omega}{c}(n_{\mathrm{eff}}(\omega)d) = \ln\left(\frac{S_{11}(\omega)}{1 - S_{21}(\omega)R(\omega)}\right), \qquad (2)$$

where c is the velocity of light in vacuum.

Since an argument of logarithm in Eq. (2) is complex there are multiple values for $n_{\mathrm{eff}}(\omega)$. If we define the argument of logarithm function on the right side of Eq. (2) as

$$F(\omega) = \frac{S_{21}(\omega)}{1 - S_{11}(\omega)R(\omega)} = |F(\omega)|\mathrm{e}^{-\mathrm{i}\phi(\omega)}, \qquad (3)$$

then $n_{\mathrm{eff}}(\omega)$ is given by

$$n_{\mathrm{eff}}(\omega) = -\frac{c}{\omega d}\{\mathrm{i}[\phi(\omega) + 2\pi n] - \ln(|F(\omega)|)\}, \qquad (4)$$

where $n = 0 \pm 1, \pm 2, \ldots$.

We consider the wavelength range $\lambda_m > d$, where λ_m is the wavelength in sample material. In this case, $n = 0$ [2].

Within the region of sample there is an effective characteristic impedance $Z_{\mathrm{eff}}(\omega) = \sqrt{\mu_{\mathrm{eff}}(\omega)/\varepsilon_{\mathrm{eff}}(\omega)}Z_0$ where Z_0 is the characteristic impedance of the air. Reflection coefficients for normal incidence of wave on the interface from air-filled outer can be given by

$$R_\perp(\omega) = -R_\parallel(\omega) = \frac{\sqrt{\mu_{\mathrm{eff}}(\omega)/\varepsilon_{\mathrm{eff}}(\omega)} - 1}{\sqrt{\mu_{\mathrm{eff}}(\omega)/\varepsilon_{\mathrm{eff}}(\omega)} + 1},$$

where $R_\perp(\omega)$ is the Fresnel coefficient of vertical polarization and $R_\parallel(\omega)$ is the Fresnel reflection coefficient of parallel polarization for the case of normal incident. Using simple manipulation we can obtain formulas to evaluate the effective constant for both the polarizations.

$$\varepsilon_{\mathrm{eff}}(\omega) = \frac{1 - R_\perp(\omega)}{1 + R_\perp(\omega)}n_{\mathrm{eff}}(\omega),$$

$$\mu_{\mathrm{eff}}(\omega) = \frac{1 + R_\perp(\omega)}{1 - R_\perp(\omega)}n_{\mathrm{eff}}(\omega), \qquad (5a)$$

$$\varepsilon_{\mathrm{eff}}(\omega) = \frac{1 + R_\parallel(\omega)}{1 - R_\parallel(\omega)}n_{\mathrm{eff}}(\omega),$$

$$\mu_{\mathrm{eff}}(\omega) = \frac{1 - R_\perp(\omega)}{1 + R_\parallel(\omega)}n_{\mathrm{eff}}(\omega). \qquad (5b)$$

As we see from formulas (4) and (5) in the case of a symmetrical metamaterial slab, we do not have to care about the structure of metamaterial. We care only about the interface reflection coefficient of metamaterial slab and S-parameters obtained theoretically or by means of experimental measurements. It is due to this fact that an impedance of homogeneous slab does not depend on its thickness.

Expressions (4) and (5) are very convenient for evaluating the effective parameters for the case of homogeneous dielectric spacing between the inhomogeneous material infill and flat boundaries of sample since the interface reflection coefficient is defined by a simple formula. The result of calculations for a flat metamaterial slab with a unit cell, as shown in Fig. 1, is plotted in Figs. 2–7. Extension of the unit cell has been done in x- and y-axes. Unit cell is presented by symmetrically spaced copper wires and some hypothetical material matrix ($\varepsilon_r = 1$, $\mu_r = 6$). Calculation of S-parameters