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Fixation times in evolutionary games with the Moran and Fermi processes

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HIGHLIGHTS

- A Moran and Fermi mixed process is proposed.
- The conditional fixation time of a co-operator with Moran rule is larger than that of Fermi.
- The unconditional fixation time of a co-operator who obtains more information is smaller.
- The larger the difference of individuals' payoff, the smaller the unconditional fixation time.

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ABSTRACT

We combined the standard Moran and Fermi process into a mixed process with two strategies *C* (co-operation) and *D* (defection). In a well-mixed population of size $N+M$, N individuals have the same update mechanism as that of Moran process, while the other M individuals have the same update mechanism as that of Fermi process. We obtain the balance equations of the conditional fixation time and unconditional fixation time. What these equations are doing is to make numerical sense for all the figures. We find that the expectation values of conditional fixation times of a single co-operator are smaller than the average values of the standard Moran and Fermi process. In addition, the conditional fixation time of a single co-operator with update rule of Moran is larger than that of Fermi when the intensity of selection is sufficiently small. The simulation results show that the unconditional fixation time of a co-operator who obtains more information is smaller. In addition, the larger the difference of individuals' payoff, the smaller the unconditional fixation time.

1. Introduction

Evolutionary game theory, which was put forward by Smith and Price (1973), has become a standard approach to study frequency dependent selection (Smith, 1982; Weibull, 1997; Hofbauer and Sigmund, 1998; Gintis, 2000; Nowak and Sigmund, 2004; Nowak, 2006, 2013). It describes the evolutionary dynamics of a well-mixed population, which is consisting of interacting individuals taking different strategies. Evolutionary game theory provides an elegant framework for studying the competition between co-operators and defectors. There is a long tradition of modeling the evolution of co-operation for evolutionary game dynamics. The traditional approach studies evolutionary game dynamics in infinitely large populations, where the stochastic effects can be neglected. This typically leads to the deterministic replicator dynamics (Hofbauer et al., 1979; Schuster and Sigmund, 1983; Hofbauer and Sigmund, 1998, 2003; Nowak et al., 2004), which is showed by nolinear differential equations. More recently, the focus has been turned to finite populations, where the stochastic approach is required (Traulsen et al., 2005; Antal and Scheuring, 2006; Taylor et al., 2006; Woelfing and Traulsen, 2009). The most popular models in finite populations are the frequency dependent Moran process (Altrock et al., 2012), and evolutionary game dynamics have also been studied in the frequency dependent Fermi process (Traulsen et al., 2006, 2007).

We focus on the following situation. The residents of a community have been asked to raise money for greening. They have a right to provide funding (co-operation) or provide nothing (defection). The money provided by every co-operator is the same. The greening degree of the community is decided by co-operators' money. There are two kinds of residents in the community. Among them, some people get information by surfing the Internet. They can catch all the news including who provide funding and how the greening is going, which is regarded as global information. Others

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get the information only by communicating with neighbors since they could not surf the Internet. They know nothing but the news of their neighbors including whether to provide funding, which is regarded as local information. This leads to a system with information asymmetry. For distinguishing between two kinds of people, in this article, two different updating rules are used to study the system with information asymmetry. Our motivation is to explore whether or not the system with information asymmetry is in favor of the formation of co-operation.

Most previous works assumed that individuals use the same updating rule, which can be considered that every individual knows information with the same level. However, in the system with information asymmetry, two kinds of people own two different levels of information. It is unreasonable to study this system with only one updating rule. For distinguishing between those kinds of people, in this article, two different updating rules are used to study the system with information asymmetry.

The heterogeneity of updating rule in a population was investigated previously for some cases (Moyano and Sanchez, 2009; Szabó et al., 2009; Szolnoki et al., 2009). For example, Szabó and Szolnoki have studied that the players who use different imitation rules can adopt the strategies and/or updating rules of their opposite players. Moyano and Sanchez have studied that if two strategies and two updating rules are allowed, agents can update both strategy and updating rule.

Our recent work has been focusing on the probability that a certain strategy takes over (Liu et al., 2015). The fixation time associated with our mixed process has received no attention so far. Here, we analyze the conditional and unconditional fixation times in evolutionary 2×2 games.

The paper is organized as follows. In Section 2, we introduce a particular evolutionary process for our analysis. We obtain the balance equations of conditional fixation times and unconditional fixation times in Section 3. We discuss the fixation times in standard Moran and Fermi process in Section 4. Section 5 offers some concrete examples. In Section 6, we give a conclusion of our results.

2. Model

We have a well-mixed population of $N+M$ individuals, each individual uses strategy either C (co-operation) or D (defection). Among these $N+M$ individuals, N individuals have the same update mechanism as that of Moran process (Allen and Nowak, 2012), while the other M individuals have update mechanism the same as that of Fermi process (Altrock and Traulsen, 2009a, 2009b). At each time step, a randomly chosen individual X evaluates its success. If its update mechanism is the same as that of Fermi process, it compares its payoff with a second, randomly chosen individual Y. The probability that it will imitate Y's strategy is given by the Fermi distribution. If its update mechanism is the same as that of Moran process, X imitates the strategy of every individual including itself with a probability proportional to every individual's fitness, such that individuals with a higher fitness are more likely to be imitated (Hauert and Doebeli, 2004; Sigmund, 2010; Wu et al., 2015). The fitness of strategy C and D is denoted by f_C and f_D, respectively. The probability to imitate individuals with strategy C is $(n+m)f_C/((n+m)f_C+(N+M-n-m)f_D)$, while the probability to imitate individuals with strategy D is $(N+M-n-m)f_D/((n+m)f_C+(N+M-n-m)f_D)$.

In the mixed process, the symmetric 2×2 game can be described by the payoff matrix:

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} a & b \\ c & d \end{pmatrix} \end{array} \tag{1}$$

The number of co-operators with update rule of Moran is given by n , and the number of co-operators with update rule of Fermi is given by m. We choose

$$f_C = e^{\omega\pi_C} \tag{2}$$

$$f_D = e^{\omega\pi_D} \tag{3}$$

where $\pi_C=(a(n+m-1)+b(N+M-n-m))/(N+M-1)$ is the average payoff of C, $\pi_D=(c(n+m)+d(N+M-n-m-1))/(N+M-1)$ is the average payoff of D, and ω is the intensity of selection.

We denote

$$\Delta\pi=\pi_C-\pi_D=u(n+m)+v \tag{4}$$

where $u=\frac{a+d-(b+c)}{N+M-1}$, $v=\frac{(N+M)(b-d)-(a-d)}{N+M-1}$.

The state of the system is characterized by (n,m). $P_{+,0}(n,m)$ and $P_{-,0}(n,m)$ represent the transition probability moving from (n,m) to $(n+1,m)$ and $(n-1,m)$, respectively, in a given time step. $P_{0,+}(n,m)$ and $P_{0,-}(n,m)$ represent the transition probability moving from (n,m) to $(n,m+1)$ and $(n,m-1)$, respectively, in a given time step. $P_{0,0}(n,m)$ denotes the probability that the population remains in state (n,m).

These transition probabilities are given by:

$$P_{+,0}(n,m)=\frac{N-n}{N+M}\frac{(n+m)f_C}{(n+m)f_C+(N+M-n-m)f_D} \tag{5}$$

$$P_{-,0}(n,m)=\frac{n}{N+M}\frac{(N+M-n-m)f_D}{(n+m)f_C+(N+M-n-m)f_D} \tag{6}$$

$$P_{0,+}(n,m)=\frac{M-m}{N+M}\frac{n+m}{N+M}\frac{1}{1+e^{\omega(\pi_D-\pi_C)}} \tag{7}$$

$$P_{0,-}(n,m)=\frac{m}{N+M}\frac{N+M-n-m}{N+M}\frac{1}{1+e^{\omega(\pi_C-\pi_D)}} \tag{8}$$

$$P_{0,0}(n,m)=1-P_{+,0}(n,m)-P_{-,0}(n,m)-P_{0,+}(n,m)-P_{0,-}(n,m) \tag{9}$$

Specially, it is noteworthy that the mixed process is identical with Fermi process when $N=0$ is satisfied, while it is identical with Moran process when $M=0$ is satisfied.

3. Fixation times

The two pure states all C (state (N,M)) or all D (state $(0,0)$) are absorbing. Unconditional fixation time $t_{n,m}$ denotes the average time until either one of the two absorbing states is reached when starting with state (n,m). Fixation has already occurred when we start in state $(0,0)$ or (N,M), thus

$$t_{0,0}=0 \tag{10}$$

$$t_{N,M}=0 \tag{11}$$

There is a balance equation for the unconditional fixation times:

$$\begin{aligned} t_{n,m} &= p_{-,0}(n,m)t_{n-1,m}+p_{0,-}(n,m)t_{n,m-1} \\ &\quad +p_{+,0}(n,m)t_{n+1,m}+p_{0,+}(n,m)t_{n,m+1} \\ &\quad +\left[1-p_{-,0}(n,m)-p_{0,-}(n,m)-p_{+,0}(n,m)-p_{0,+}(n,m)\right]t_{n,m}+1 \end{aligned} \tag{12}$$