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Determination of spatially dependent diffusion parameters in bovine bone using Kalman filter

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ABSTRACT

Although many studies have been made for homogenous constant diffusion, bone is an inhomogeneous material. It has been suggested that bone porosity decreases from the inner boundaries to the outer boundaries of the long bones. The diffusivity of substances in the bone matrix is believed to increase as the bone porosity increases. In this study, an experimental set up is used where bovine bone samples, saturated with potassium chloride (KCl), were put into distilled water and the conductivity of the water was followed. Chloride ions in the bone samples escaped out in the water through diffusion and the increase of the conductivity was measured. A one-dimensional, spatially dependent mathematical model describing the diffusion process is used. The diffusion parameters in the model are determined using a Kalman filter technique. The parameters for spatially dependent at endosteal and periosteal surfaces are found to be $(12.8 \pm 4.7) \times 10^{-11}$ and $(5 \pm 3.5) \times 10^{-11}$ m^2/s respectively. The mathematical model function using the obtained diffusion parameters fits very well with the experimental data with mean square error varies from 0.06×10^{-6} to 0.183×10^{-6} $(\mu S/m)^2$

1. Introduction

Diffusion can be defined as the process by which matter is transported from one part of a system to another as a result of random molecular motion (Crank, 1975). The commonly used mathematical representation of the diffusion process that can be used to find the diffusion parameter is the so-called Fick's law. Based on this mathematical representation, many researchers were able to derive and use models based on diffusion in many different fields (Cayan et al., 2009; Margetis, 2009; Naceri, 2009; Yildirim et al., 2011).

One of the fields where Fick's laws can be applied is for representing diffusion in blood cell membranes. More specifically, such diffusion involves the diffusion of nutrients and oxygen from high concentrations in the blood vessels to less concentrations in the blood cells. For example, models including diffusion, have been presented in studies of bone healing where bone is repaired after fracture (Adam, 2002; Ambard and Swider, 2006; Chou et al., 2013; Gomez-Benito et al., 2005; Sapotnick and Nackenhorst, 2012). Other models have been presented for bone remodeling, in which old bone tissue is replaced by a new one (Adachi et al., 2006; Fernández et al., 2012; Stadelmann et al., 2009). Stress or strain driven diffusion have been suggested to play an important role in the transportation of substances from the inner surface, endosteal, that covers medullar canal in the long bone to the outer, periosteal, bone surface. A one-dimensional model for strain driven transport of bone nutrients has been presented by Banks-Sills et al. (2011). This study was followed up by another study (Lindberg et al., 2013) where a two-dimensional model describing the stress driven diffusion was presented.

Several studies have investigated the bone structure and the apparent diffusion coefficient using magnetic resonance imaging (Ababneh et al., 2009; Balliu et al., 2009; Capuani, 2013). The latter method is very suitable for obtaining information about the structural properties of the bone. A less complex method for determining the diffusion coefficient has been introduced by Lindberg et al. (2014). They used an experimental set up where the increased conductivity in distilled water due to the diffusing chloride ions escaping from saturated bone samples was measured over time. Further, the diffusion parameter was determined from fitting the parameters in an analytical model according to the experimental data using a Kalman filtering technique. Kalman filters could be used as a recursive method in order to extract unknown parameters from noisy experimental measurements. In previous studies, despite the very complex structure of bone, the diffusion parameter was assumed to be independent of the position in the bone wall. The diffusion of solutes that promote

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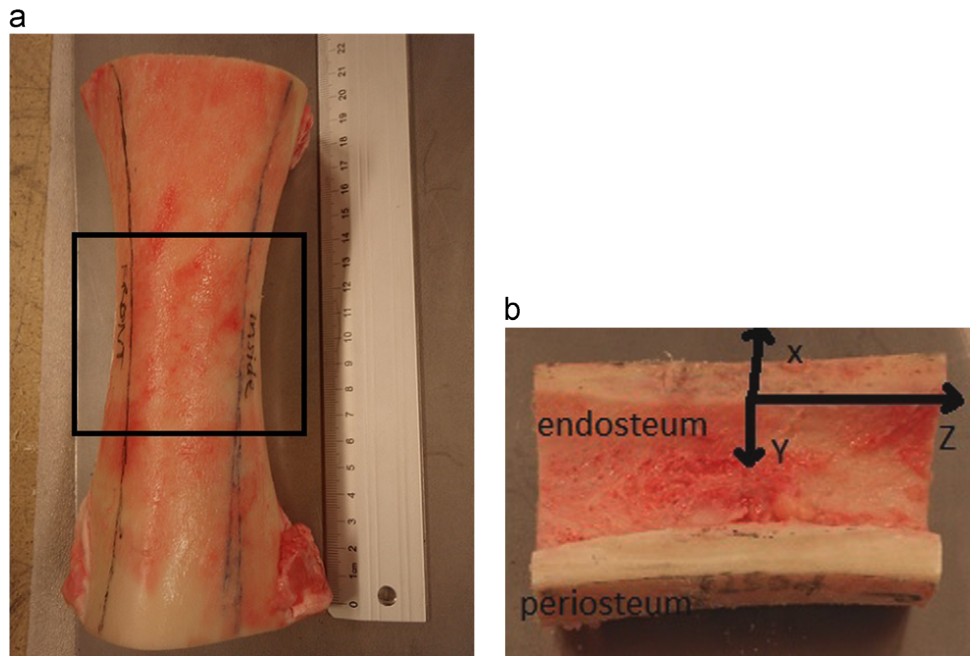

Fig. 1. Images of the Holstein-cattle femur bone, (a) long bone with a black rectangle indicates position of samples, (b) a midsection of the diaphysis cut along the long axis showing the endosteum and periosteum. Note the incorporated coordinate system in the picture.

bone growth are crucial for creating a good environment for bone remodeling and bone healing. Therefore, it is important to get a better understanding of the diffusion process and determine the parameters involved. Bone has been found to have hierarchically decreasing porosity from inner boundaries to outer boundaries of cortical bone (Cowin, 1999; Cowin et al., 2009). Since it is believed that diffusivity increases as bone porosity increases, the observed variation in bone porosity supports the hypothesis that the diffusion parameter may decrease linearly from inner boundaries to outer boundaries of bone.

In this study, a one-dimensional model for linear diffusion in the radial direction, assuming linear spatially dependent diffusivity will be presented. Dominantly, the diffusion of nutrients and signal substances are transported in the radial direction from the marrow and blood vessels to the periosteal bone surface. Two diffusion parameters for the bone sample based on the new model will be determined from experimental results using a Kalman filter technique.

2. Materials and methods

2.1. Experimental setup

The diffusion of potassium chloride in the mid-shaft of a long bovine bone was studied. The samples were taken from the central part between the ends of a femur bone of a Holstein-cattle (marked with black rectangle in Fig. 1a) at scattered angular positions around the bone cross-section, since previous research has found that the position (anterior, posterior, medial, lateral) around the periphery of the bone has no significant effect on the diffusion properties (Lindberg et al., 2014).

The femur bones were cut into pieces according to Fig. 1b, cut free from flesh and quickly rinsed in running water. The bone pieces were then cut into smaller samples and again quickly rinsed in water. The dimensions of the bone samples are given in Table 1. Note that the size in the radial direction, L, corresponds to the wall thickness of the bone.

Table 1
Sample dimensions in mm. L, h and b correspond to the sizes in radial, tangential and axial directions, in respectively.

Sample	1	2	3	4	5
L (mm)	7.0	9.05	7.9	6.6	7.0
h (mm)	19.7	21.2	20.5	22.5	20.1
b (mm)	13.6	16.1	16.5	14.3	15.3

The samples were stored at around -4 °C in a freezer for a few days up to a couple of weeks. Just before the measurements they were placed in room temperature for one hour. The samples were then submerged into polyester resin for a curing time of one hour. The endosteal surface and the periosteal surface were covered by a layer of rubber, after the curing the rubber was peeled off. Thus, the surfaces with axial and tangential normal directions were sealed, leaving only the periosteal and endosteal surfaces open as shown in Fig. 2.

The samples were then put in a saturated potassium chloride, KCl, solution for 24 h, which is supposed to be sufficiently long for the KCl ions to diffuse into the samples so that a fairly constant concentration is achieved (see Appendix A for details). After flushing in distilled water for around 5 s, the samples were finally submerged in 100 ml of distilled water which allows the potassium and chloride ions to diffuse through the bone to the surface where they escape into the surrounding water. The conductivity in the surrounding water was measured once per second using a SevenEasy S30 conductivity meter from Mettler Toledo. The instrument has an accuracy of $\pm 0.5\%$ of the measured value. The equipment was calibrated at 25 °C but during operation (presumably due to stirring) the temperature rose a couple of centigrades but remained in the interval 28 ± 3 °C.

2.2. Analytical model

Bousson et al. (2001) and Baron (2012) studied the porosity of human long bones from females of different age. The measurements were taken at three different positions in the bone wall: at