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Evolutionary dynamics of synergistic and discounted group interactions in structured populations

Aming Li [a,b,*], Long Wang [a]

[a] Center for Systems and Control, College of Engineering, Peking University, Beijing 100871, China
[b] Center for Complex Network Research and Department of Physics, Northeastern University, Boston, MA 02115, USA

HIGHLIGHTS

- Evolutionary dynamics is given in infinite structured populations.
- The dynamics contains synergistic, linear, and discounted group interactions.
- Five classical scenarios of evolutionary outcomes are recovered by the dynamics.
- Linear spatial group interactions could not give much complexity in the dynamics.
- Synergistic group interactions are not always beneficial for cooperation.

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ABSTRACT

The emergence of cooperation between unrelated individuals enables researchers to study how the collective cooperative behavior survives in a world where egoists could get more short-term benefits. The spatial multi-player games, which invoke interactions between individuals who are not directly linked by the interactive networks, are drawing more and more attention in exploring the evolution of cooperation. Here we address the evolutionary dynamics in infinite structured populations with discounted, linear, and synergistic group interactions. The five classical scenarios are recovered from the dynamics: (i) dominating defection, (ii) dominating cooperation, (iii) co-existence, (iv) bi-stability, and (v) neutral variants. For linear interactions, the evolutionary dynamics is equivalent to that in finite as well as the well-mixed counterparts, which can be achieved by a payoff matrix transformation, and it illustrates that the more neighbors there are, the harder the cooperators survive. Yet both cooperation and defection emerge easier in finite populations than in infinite for discounted and synergistic interactions. Counterintuitively, we find that the synergistic group interactions always raise cooperators' barriers to occupy the population with the increase of the number of neighbors in infinite structured populations. Our results go against the common belief that synergistic interactions are necessarily beneficial for the cooperative behavior.

1. Introduction

The evolution of the cooperative behavior in a competitive world ranging from multicellular organisms to human societies is an enduring conundrum (Hamilton, 1963, 1964; Smith, 1964; Hardin, 1968; Trivers, 1971; Dugatkin, 1997; Nowak, 2006a; Pennisi, 2009; Traulsen et al., 2010; Rand and Nowak, 2013). Why should a cooperator pay a cost to benefit another individual who may become a competitor in the future struggle for survival? As a prominent metaphor, many types of games (von Neumann and Morgenstern, 1944; Axelrod, 1984; Rapoport and Chammah, 1965; Sugden, 1986; Skyrms, 2004) are employed to deal with the problem. And some mechanisms have been put forward successively to support the evolution of cooperation (Hamilton, 1964; Axelrod, 2012; Nowak and May, 1992; Trivers, 1971; Nowak and Sigmund, 1998; Traulsen and Nowak, 2006; Traulsen et al., 2008; Nowak, 2006b). The mechanisms causing cooperation to be facilitated over defection determine the interactive patterns of the individuals to acquire payoffs and also the competition between them (Nowak, 2006b; Rand and Nowak, 2013; Wu et al., 2009; Hauert et al., 2002a; Fu et al., 2008; Wang et al., 2006; Yang et al., 2013). According to the various interactive patterns, the frequency of different strategies

* Corresponding author.
E-mail addresses: amingli2011@gmail.com (A. Li), longwang@pku.edu.cn (L. Wang).

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changes over time in populations. The classical methods adopted to capture the changes are replicator equation in infinite populations (Taylor and Jonker, 1978; Hofbauer and Sigmund, 1998), stochastic dynamics in finite populations (Nowak et al., 2004; Traulsen et al., 2005), and Monte Carlo simulations in structured populations (Szabó and Fáth, 2007; Perc and Szolnoki, 2010; Perc and Gómez-Gardeñes, 2013).

The replicator equation is a fundamental equation of evolutionary dynamics. It depicts the frequency dependent selection assuming implicitly infinite well-mixed populations, where strategies with higher average payoffs become more common (Taylor and Jonker, 1978; Hofbauer and Sigmund, 1998). The structure of a well-mixed (also can be regarded as structureless) population can be represented by a complete graph (or network) with vertices indicating players and edges indicating social ties (Lieberman et al., 2005; Ohtsuki et al., 2006; Ohtsuki and Nowak, 2006). Recently, the prominent metaphors of the one-shot, symmetric two-player dilemmas of cooperation such as the Prisoner's Dilemma (Rapoport and Chammah, 1965), Snowdrift Game (Sugden, 1986), and Stag Hunt Game (Skyrms, 2004) have been extended to their N-player versions (Broom et al., 1997; Hauert et al., 2002b; Bach et al., 2006; Zheng et al., 2007; Santos et al., 2008, 2012; Souza et al., 2009; Gokhale and Traulsen, 2010; Pacheco et al., 2009; Wu et al., 2013), a closer situation to the natural society, by which researchers could investigate the collective behavior. These extensions open the possibility for the appearance of new interior fixed points in the replicator equation (Zheng et al., 2007; Gokhale and Traulsen, 2010; Santos et al., 2012). Furthermore, the well-mixed population structure, an ideal case in which every player interacts with all others in the same population at an identical probability, has also been removed (Santos et al., 2008, 2012; Rand et al., 2011; Perc, 2011; Perc and Gómez-Gardeñes, 2013; Li et al., 2014; Frean et al., 2013; Chen and Wang, 2010; Liu et al., 2013).

Since the seminal work conducted by Santos et al. (2008) on complex networks, many interesting results have been obtained on games governed by group interactions in structured populations. Many factors, such as group size (Szolnoki and Perc, 2011), payoff distributions (Perc, 2011), initial investments (Gao et al., 2010; Vukov et al., 2011; Huang et al., 2015), teaching activity (Guan et al., 2007), group reputation (Li et al., 2013), and individual expectation (Wu et al., 2012; Li et al., 2013; Wang et al., 2014), and various heterogeneous networks with degree correlations (Rong and Wu, 2009), highly clustering (Rong et al., 2010), and uncorrelated relations (Yang et al., 2009) receive much attention in the context of group interactions in structured populations (see Perc and Gómez-Gardeñes, 2013 for a more thorough exposition). The spatial multiple interactions are more than just the corresponding sum of pairwise interactions (Szolnoki et al., 2009; Perc and Gómez-Gardeñes, 2013), and it invokes effective connections between players who are not linked directly by means of the interactive networks (Santos et al., 2008, 2012; Szolnoki et al., 2009; Szolnoki and Perc, 2011; Li et al., 2013; Perc and Gómez-Gardeñes, 2013; Li et al., 2014; Frean et al., 2013; Débarre et al., 2014; Pinheiro et al., 2014). With numerical simulations, Santos et al. investigate the evolution of cooperation in finite structured populations with the N-player snowdrift game, where the public goods are produced only when the number of the cooperators sharing the required workload is bigger than the minimum threshold (Santos et al., 2012). And they find that the homogeneous population structure enhances the chances of coordinating toward stable levels of cooperation. In finite structured populations with group interactions, Li et al. give a rule which determines the emergence and stabilization of cooperation theoretically (Li et al., 2014). They find that the synergistic interactions could change the scenario in public goods games, under which the intermediate number of neighbors is the worst case for the survival of cooperators.

However, from the theoretical aspect, as individuals are engaged in multiple interactions, how the population structure or the number of neighbors affects the evolution of cooperation in infinite structured populations is still unclear. Here, using pair approximation, we investigate the evolution of cooperation in infinite structured populations with discounted, linear, and synergistic group interactions. As to the group interactions, the additional cooperators often contribute a higher efficiency than linear increase to the real situation, such as the threshold effects and other nonlinear interactions (Gore et al., 2009; Hauert et al., 2006; Archetti and Scheuring, 2011, 2012; Santos et al., 2012; Souza et al., 2009; Santos and Pacheco, 2011). We adopt discounted and synergistic effects as an example to indicate the nonlinearity in public goods here, considering the mathematical convenience and the concept of synergy and discounting can be used to unify the prisoners dilemma and snowdrift game (Hauert et al., 2006). For the infinite structured populations, we will not be surprised if we observe the widespread tremendous network structure in human organizations (Skyrms and Pemantle, 2000; Jackson and Watts, 2002), scientific collaboration in researchers (Newman, 2001), and also the movie actors (Borgatti et al., 2009; Barabási, 2002; Barabási and Albert, 1999).

2. Evolutionary dynamics

Let us consider an infinite structured population depicted by a regular graph with degree k, where vertices indicate individuals and edges social ties. The framework of the public goods game including discounting and synergy effects is adopted to capture the discounted, linear, and synergistic group interactions (Hauert et al., 2006; Li et al., 2014). In a group of size n with i cooperators, the defectors and cooperators receive the following payoffs:

$$P_D(i) = \frac{b}{n}(1+\delta+\delta^2+\cdots+\delta^{i-1}) = \frac{b}{n}\frac{1-\delta^i}{1-\delta}$$
$$P_C(i) = P_D(i) - c \tag{1}$$

where the jth (in a general way, $1 \le j \le n$) cooperator pays a cost c for all of the members within the same group to receive the benefit $b\delta^{j-1}/n$ equally while defectors pay nothing, and δ is the discounting ($0<\delta<1$) and synergy ($\delta>1$) factor. As $\delta=1$, $P_D(i)=bi/n$, the game reduces to the linear public goods game (Archetti and Scheuring, 2012) with $P_D(i)=rci/n$, and in this case, $b=rc$, where r represents the multiplication factor of the common pool.

Different from the well-mixed population, our model allows players to interact locally, i.e., every individual participates in only k games organized by its neighbors and one additional game by itself (Santos et al., 2008; Li et al., 2014). Exactly, it is not just the sum of the corresponding pairwise interactions in terms of players' payoffs. As to the evolutionary dynamics, the "death–birth" (DB) process is employed to capture the update process, where an individual in the population is randomly chosen to die at each evolutionary step, and then all of its neighbors compete for the vacant site proportional to their fitness (Nowak, 2006a; Ohtsuki et al., 2006).

Using the framework of pair approximation, in every elementary step of updating, we get the expected change of the frequency of cooperators indicated by x in infinite structured populations with weak selection w (see Appendix A). Furthermore, we obtain the deterministic evolutionary dynamics

$$\dot{x} = w\frac{k-2}{k(k-1)}x(1-x)f(x) \tag{2}$$

where

$$f(x) = (k-2)(\delta^2-1)b\delta^{(k-2)x}x - k(k+1)c + \frac{(k+1)\delta^2+2k\delta+k^2-1}{k+1}b\delta^{(k-2)x}. \tag{3}$$