Journal of Theoretical Biology 341 (2014) 53–63

Contents lists available at ScienceDirect

Journal of Theoretical Biology

journal homepage: www.elsevier.com/locate/yjtbi

Finite element analysis of a femur to deconstruct the paradox of bone curvature

Sameer Jade [a], Kelli H. Tamvada [b], David S. Strait [b], Ian R. Grosse [a,*]

[a] Department of Mechanical and Industrial Engineering, 160 Governor's Drive, University of Massachusetts, Amherst, MA 01003, USA
[b] Department of Anthropology, Arts and Sciences 237, 1400 Washington Ave., University of Albany, NY 12222, USA

HIGHLIGHTS

- FEA was used to study structural strength and bending predictability in long bones.
- Load carrying capacity can be compromised by bone curvature.
- Load carrying capacity can be also be increased by bone curvature.
- Curvature does increase bending predictability.
- Probability density functions can be generated for bending predictability.

ARTICLE INFO

Article history:
Received 22 October 2012
Received in revised form
6 September 2013
Accepted 11 September 2013
Available online 4 October 2013

Keywords:
FEA
Bone curvature
Bending predictability

ABSTRACT

Most long limb bones in terrestrial mammals exhibit a longitudinal curvature and have been found to be loaded in bending. Bone curvature poses a paradox in terms of the mechanical function of limb bones, for many believe the curvature in these bones increases bending stress, potentially reducing the bone's load carrying capacity (i.e., its mechanical strength). The aim of this study is to investigate the role of longitudinal bone curvature in the design of limb bones. In particular, it has been hypothesized that bone curvature results in a trade-off between the bone's mechanical strength and its bending predictability. We employed finite element analysis (FEA) of abstract and realistic human femora to address this issue. Geometrically simplified human femur models with different curvatures were developed and analyzed with a commercial FEA tool to examine how curvature affects the bone's bending predictability and load carrying capacity. Results were post-processed to yield probability density functions (PDFs) describing the circumferential location of maximum equivalent stress for various curvatures in order to assess bending predictability. To validate our findings, a finite element model was built from a CT scan of a real human femur and compared to the simplified femur model. We found general agreement in trends but some quantitative differences most likely due to the geometric differences between the digitally reconstructed and the simplified finite element models. As hypothesized by others, our results support the hypothesis that bone curvature can increase bending predictability, but at the expense of bone strength.

1. Introduction

In a classic study, John Bertram and Andrew Biewener examined the mechanics of bone curvature in the design of the long (i.e., limb) bones of mammals based on elementary analytical expressions (Bertram and Biewener 1988). They noted that although a distinct longitudinal curvature was ubiquitous in mammals, a straight bone should be a more efficient design because the maximum mechanical strength of a straight bone should be higher. Bertram and Biewener hypothesized that the benefit of bone curvature is that it makes more predictable the manner in which the bone bends. Bending predictability was defined by Bertram and Biewener as the probability of the bone to bend in a certain direction, thereby yielding consistent stress patterns within the bone. In other words, curvature restricts the range of bending directions when a bone is subjected to loads in variable directions. In contrast, a straight bone subjected to the same loads will have no restriction on the range of its bending direction and hence will have no bending predictability (Bertram and Biewener, 1988). Although other explanations for bone curvature have been proposed (Bertram and Biewener, 1988; Yamanaka et al., 2005; Taylor et al., 1996; Rubin and Lanyon, 1982; Biewener, 1983; Biewener, 1986; Frost, 1979; Lanyon, 1980), this study focuses

* Corresponding author. Tel.: +1 413 545 1350; fax: +1 413 545 1027.
E-mail address: grosse@ecs.umass.edu (I.R. Grosse).

0022-5193/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.jtbi.2013.09.012

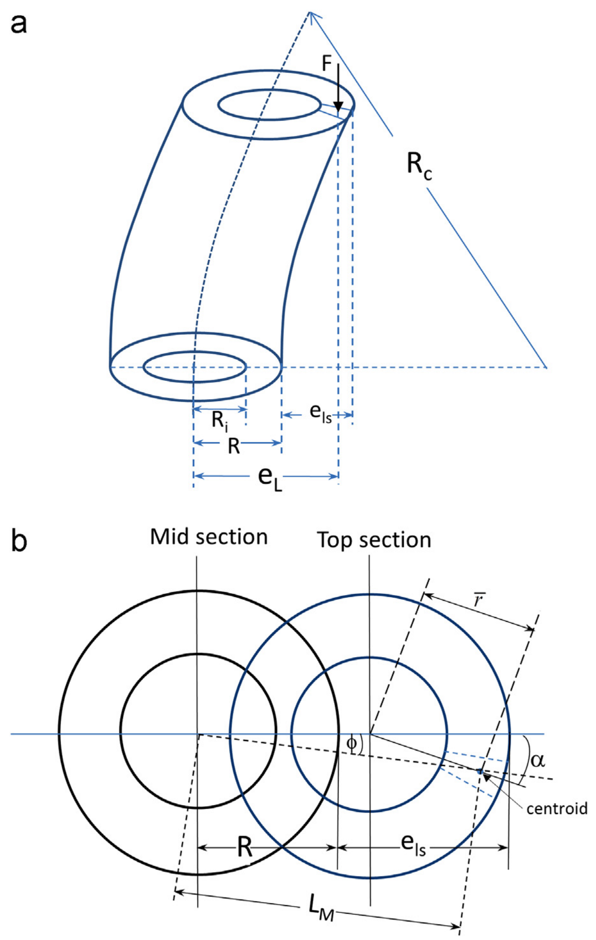

Fig. 1. (a) A simplified model of a curved bone loaded axially on its outer edge by a load f showing longitudinal shape eccentricity (e_{ls}), load eccentricity (e_L) and radius of curvature (R_c). The inferior section shown is assumed to be a plane of symmetry. (b) Top view of the top section and midsection of the bone showing the moment arm, L_m. The angle α defines the circumferential location of the axial load. Note that as alpha increases, L_m decreases which decreases the bending moment. This is reflected in Fig. 10 with higher *RLCC* occurs at higher alpha values.

on the predictability hypothesis. Predictions of this hypothesis are tested using finite element analysis (FEA), a modeling method widely employed in engineering that can be used to examine how objects of complex design respond to load. Here, we use FEA to determine how bone curvature affects its bending predictability, as well as its strength, and compare these results to those obtained using the analytical model employed by Bertram and Biewener. A review of Bertram and Biewener's analytical model is first presented.

1.1. The Bertram and Biewener model revisited

Fig. 1 shows the geometry of the bone used for the derivation. Bertram and Biewener and other researchers (Bertram and Biewener, 1988; Yamanaka et al., 2005; Biewener, 1983) in the past have used the term bone curvature, c, to define what we call in Fig. 1 the bone's longitudinal shape eccentricity, e_{ls}, in this study. We propose this change in terminology since curvature is mathematically defined as the reciprocal of the radius of curvature, and the parameter e_{ls} shown in Fig. 1 is clearly not the reciprocal of the radius of curvature of the bone. The bone's longitudinal shape eccentricity, e_{ls}, is not to be confused with the traditional definition of a bone's shape eccentricity which is based on its cross section; this should be correctly termed the bone's cross-sectional shape eccentricity. In general, long bones have two eccentricities in shape: in the longitudinal direction and in cross-section. The former refers to the deviation of the longitudinal axis of the bone from a straight line, while the latter refers to the deviation of the bone's cross section from a circular shape. The mathematical relationship between a true bone curvature, c, and its radius of curvature, R_c with its longitudinal shape eccentricity, e_{ls}, for a bone of circular cross section is given by:

$$\frac{R_c}{R} = \frac{1}{cR} = \frac{4(e_{ls}/R)^2 + (L/R)^2}{8(e_{ls}/R)}. \tag{1}$$

where L is the length of the bone and R is the radius of the bone's section. In Fig. 2 we plot the radius of curvature of bone and bone's true curvature against the longitudinal shape eccentricity of the bone using Eq. (1) for a range of longitudinal shape eccentricities found in long limb bones of humans (Schonning et al., 2009). The radius of curvature of the bone is inversely proportional to the longitudinal shape eccentricity of the bone, and the true bone curvature increases monotonically with the longitudinal shape eccentricity. In Fig. 1 we also define the eccentricity of the load as e_L. Load eccentricity is defined as the perpendicular distance of the load from the center of the bone section and is maximum at the bone's longitudinal mid-plane for our idealized bone model.

Following Bertram and Biewener (Bertram and Biewener, 1988), let $f_{\max}$ be the maximum load supported by a curved bone, i.e., the load that produces a total normal stress in the bone equal to the yield strength of the bone, S_y. For the case of an axial load in the plane of curvature of the bone (i.e., $\alpha = 0^\circ$), the maximum normal compressive stress produced in a bone of geometry defined as in Fig. 1 occurs on the longitudinal mid-plane on the posterior surface and is given by

$$\begin{aligned} S_y = \sigma_{\max} &= \frac{f_{\max}}{A} + \frac{M_{\max}R}{I} = \frac{f_{\max}}{\pi R^2} + \frac{4f_{\max}(R+e_{ls})R}{\pi R^4} \\ S_y = \sigma_{\max} &= f_{\max}\left(\frac{R^2 + 4(R+e_{ls})R}{\pi R^4}\right) \end{aligned} \tag{2}$$

where $M_{\max}$ is the maximum bending moment, A is the area of the bone's cross section, and I is the area moment of inertia of the bone's section. Let $F_{\max}$ be the maximum load supported by a straight bone for which $e_{ls} = 0$:

$$S_y = \sigma_{\max} = F_{\max}\left(\frac{R^2 + 4R^2}{\pi R^4}\right) = F_{\max}\left(\frac{5}{\pi R^2}\right). \tag{3}$$

Equating Eqs. (2) and (3) and solving for the ratio of $f_{\max}/F_{\max}$ which is the relative load carrying capacity (*RLLC*) the bone yields,

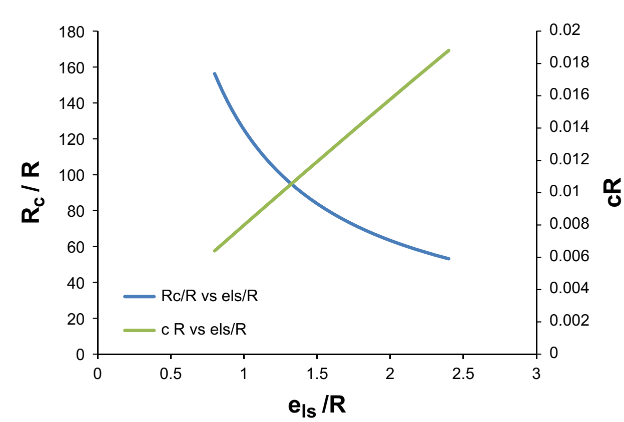

Fig. 2. Radius of Curvature R_c and Curvature c vs. Longitudinal Shape Eccentricity e_{ls}. Note that R_c is non-dimensionalized by the radius of the bone's cross section R (R_c/R) and curvature c and longitudinal shape eccentricity e_{ls} are non-dimensionalized by the length of the bone L (cL and e_{ls}/L).