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Random diffusion and cooperation in continuous two-dimensional space

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HIGHLIGHTS

- We model population games with agent mobility in continuous space.
- Random mobility favors cooperation when agents imitate their most successful neighbor and move with constant velocity.
- When agents move in a viscous medium velocity decreases and stable monomorphic clusters form.

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ABSTRACT

This work presents a systematic study of population games of the Prisoner's Dilemma, Hawk–Dove, and Stag Hunt types in two-dimensional Euclidean space under two-person, one-shot game-theoretic interactions, and in the presence of agent random mobility. The goal is to investigate whether cooperation can evolve and be stable when agents can move randomly in continuous space. When the agents all have the same constant velocity cooperation may evolve if the agents update their strategies imitating the most successful neighbor. If a fitness difference proportional is used instead, cooperation does not improve with respect to the static random geometric graph case. When viscosity effects set-in and agent velocity becomes a quickly decreasing function of the number of neighbors they have, one observes the formation of monomorphic stable clusters of cooperators or defectors in the Prisoner's Dilemma. However, cooperation does not spread in the population as in the constant velocity case.

1. Introduction and previous work

Cooperative behavior is socially beneficial but difficult to obtain among selfish individuals. In this context, the Prisoner's Dilemma game is a widely used paradigm for the investigation of how cooperation might evolve in a population of self-regarding agents. In fact, game-theoretical results predict defection as a Nash equilibrium or as a stable state of the population dynamics (Hofbauer and Sigmund, 1998; Vega-Redondo, 2003). In spite of this, non-negligible amounts of cooperative behavior can be observed daily in the animal kingdom, in the human society, and also in the laboratory, where controlled experiments can be carried out. Many mechanisms have been suggested to explain these behaviors, such as direct and indirect reciprocity, kin reciprocity, group reciprocity, and population structure among others (see e.g. Nowak, 2006 and references therein for a summary of this vast amount of work).

Among the various reasons that have been advocated, the structure of the interacting population is one of the simplest factors that can change the generalized defection outcome with respect to the well-mixed population case. The population structure of the interacting agents can be generically represented by a relational graph in which two directly linked vertices stand for two interacting agents. This locality of contacts means that only pairs or groups of individuals that are direct neighbors play the game among themselves. By using theoretical models and simulations, it has been found that some network structures appear to be more conducive to cooperation than others, albeit this result is contingent upon the evolutionary dynamics of the model (Szabó and Fáth, 2007; Santos et al., 2006a; Roca et al., 2009; Ohtsuki et al., 2006). However, an earlier way of considering the effect of population structures makes use of the concept of geographical space. Indeed, physical space may be more adequate than generic relational structures in many cases in which territoriality plays an important role. A simple first approximation of physical space is given by a regular discrete lattice in two dimensions. Building on previous work by Axelrod (1984), Nowak and May (1992) and Nowak et al. (1994) was able to show by extensive simulations that, even when the game is one-shot, i.e. pairs of players interact anonymously, cooperation can evolve and can persist for a non-negligible region of the game phase space thanks to positive

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assortment among cooperators. Of course, anonymity of neighbors is difficult to maintain in a real unchanging social network environment, but this is the context that has been adopted in previous modeling work. A summary of this and other early work is provided in Nowak and Sigmund (2000). Actually, the gains in the PD are relatively limited and depend on the players' strategy update rule used (Roca et al., 2009). Meanwhile, the improvements are large in the related game called Stag Hunt (SH) (Roca et al., 2009; Skyrms, 2004) when played on a grid. Evolutionary games on arbitrary static spatially embedded networks have been recently studied in Buesser and Tomassini (2012).

All the above refers to static environments. However, it is easy to see that fixed environments are the exception rather than the rule. Evolutionary games on dynamic networks have been investigated in recent years, see e.g. Eguíluz et al. (2005); Santos et al. (2006b); Pestelacci et al. (2008); Wu et al. (2010) and the review article (Perc and Szolnoki, 2010). Although the models differ in their assumptions and the details of the dynamics, there is a consensus emerging on the fact that purposeful, or strategic link update is a further factor allowing cooperating individuals to escape exploiting defectors by cutting links to them and forming new links with fellow cooperators, which facilitates clustering and positive assortment of cooperators, ultimately leading to sustained global cooperation. In a way analogous to the dynamic network case, in the case of spatially embedded agents it is easy to think of mobile rather than fixed individuals. Many examples can be found in biological and ecological sciences, in human populations, and in engineered systems such as ad hoc networks of mobile communicating devices or mobile robot teams. Mobility may have positive or negative effects on cooperation, depending on several factors. An early brief investigation of random grids and spatial drift is to be found in Nowak et al. (1994). Another study was carried out by Enquist and Leimar (1993) whose main conclusion of Enquist and Leimar (1993) was that mobility may seriously restrict the evolution of cooperation. In the last decade there have been several new studies of the influence of mobility on the behavior of various games in spatial environments covering essentially two strands of research: one in which the movement of agents is seen as a random walk, and a second one in which movement may contain random elements but it is purposeful, or strategy-driven. Examples of the latter kind of work are to be found in Helbing and Yu (2009), Jiang et al. (2010), Cong et al. (2012), Chen et al. (2011), Aktipis (2004) and Roca and Helbing (2011). In spite of the difference among the proposed models, the general message of this work is that purposeful contingent movement may lead to highly cooperating stable or quasi-stable population states depending on the individuals' density and the degree of mobility.

As said above, the other line of investigation is centered on random diffusion of the mobile agents through space, either in continuous space (Meloni et al., 2009) or, more commonly, on diluted grids (Vainstein et al., 2007; Sicardi et al., 2009). Random diffusion, with its tendency to mix-up the population, has been thought to hinder cooperation by weakening the possibility of cooperator cluster formation. In spite of this, the work of Vainstein et al. (2007) and Sicardi et al. (2009) shows that cooperation can be maintained with respect to the static case and even enhanced for some parameters' ranges. In the continuous space case of Meloni et al. (2009) cooperation can be maintained only for low velocities and low temptation to defect. Within this framework, there has also been work on n-person Prisoner's Dilemma and public goods games, either in the one-shot case (Cardillo et al., 2012), as well as in the iterated, short memory case (Chiong and Kirley, 2012). The effect of diffusion in a spatial ecological public goods game has been studied by Wakano et al. (2009) using a partial differential equation formalism.

The present investigation belongs to the random diffusion category and deals with memoryless agents performing random movements and interacting in pairs with other agents in continuous space. Indeed, we believe that while grids are interesting because of their simplicity, a continuous space approach is more natural and less restricted. Our approach follows Meloni et al. (2009) but it largely extends and completes it in various ways. Indeed, Meloni et al. studied the weak Prisoner's Dilemma, which is the segment at the frontier between the genuine Prisoner's Dilemma game space and the Hawk–Dove game (Hofbauer and Sigmund, 1998). Here we explore the full conventional Prisoner's Dilemma space and also the regions belonging to the Stag Hunt and Hawk–Dove games. Furthermore, we use a second strategy update rule besides their fitness-proportional one. Finally, while the velocity of the agents was held constant and the same for all individuals in the population in Meloni et al. (2009), we explore the effects of having players diffusing with different velocities. Some relationships with the results found in the grid-based diffusion systems proposed in Vainstein et al. (2007) and Sicardi et al. (2009) will also be discussed.

2. Model description

In this section we describe our model and the numerical simulations parameters. We also describe what is new with respect to the previous work.

2.1. The spatial environment

The environment in which the set of agents N interact and move is a square in the Euclidean plane of side $L=1$ thus having unit area. For the purposes of the dynamics the square is wrapped around into a torus. Agents are initially distributed uniformly at random over the space. Every agent j has an interaction neighborhood which has the same extension for all agents and is given by a circle of radius r around the given agent. All the agents that fall into this circle at a given time t are considered to be neighbors $\mathcal{N}(j,t)$ of the agent, i.e. $\mathcal{N}(j,t)=\{\forall k \in N | dist(j,k) < r\}$, where $dist(j,k)$ is the Euclidean distance between agents (points) j and k. Agents are simply material points, they do not have an area. Since the spatial region area has unit value, the density ρ of the agents is $\rho = |N|$.

Given the above setting, at any point in time the current implicit network of contacts between the agents turns out to be a Random Geometric Graph (RGG) (Dall and Christensen, 2002) as illustrated in Fig. 1. The average degree $\bar{k}$ of a RGG is $\bar{k} = \pi\rho r^2$. Thus it is possible to consider $\bar{k}$ as a parameter of RGGs, instead of the radius r. Therefore, in order to construct a RGG with an average degree that tends to $\bar{k}$, it is sufficient to use the radius $r=\sqrt{\bar{k}/(\pi\rho)}$. This class of networks has an high average clustering coefficient (Dall and Christensen, 2002) and positive degree–degree correlations (Antonioni and Tomassini, 2012).

2.2. Games studied

Agents in our system, when they interact in pairs, play one of three common two-person, two-strategy, symmetric game classes, namely the Prisoner's Dilemma (PD), the Hawk–Dove Game (HD), and the Stag Hunt (SH). These three games are simple metaphors for different kinds of dilemmas that arise when individual and social interests collide. The games have the generic payoff matrix M (Eq. (1)) which refers to the payoffs of the row player. The payoff matrix for the column player is simply the transpose $M^\top$ since the