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# A mathematical analysis of public avoidance behavior during epidemics using game theory

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ABSTRACT

The decision of individuals to engage in public avoidance during epidemics is modeled and studied using game theory. The analysis reveals that the set of Nash equilibria of the model, as well as how the equilibria compare to the social optimum, depend on the contact function that governs the rate at which encounters occur in public. If the contact ratio – defined to be the ratio of the contact rate to the number of people out in public – is increasing with the number of people out in public, then there exists a unique Nash equilibrium. Moreover, in equilibrium, the amount of public avoidance is too low with respect to social welfare. On the other hand, if the contact ratio is decreasing in the number of people out in public, then there can be multiple Nash equilibria, none of which is in general socially optimal. Furthermore, the amount of public avoidance in equilibrium with a decreasing contact ratio is too high in that social welfare can be increased if more susceptible individuals choose to go out in public. In the special case where the contact ratio does not vary with the number of people out in public, there is a unique Nash equilibrium, and it is also the socially optimal outcome.



## 1. Introduction

Recent modeling studies have shown that social distancing, which refers to measures taken by individuals or communities to reduce contacts between people, can be highly effective at curbing the spread of infectious diseases such as pandemic influenza (Caley et al., 2008; Davey et al., 2008; Glass et al., 2006; Halloran et al., 2008; Kelso et al., 2009; Milne et al., 2008). Naturally, the extent to which social distancing can contain an epidemic depends on the specific policies that communities choose to adopt and the incentives that individuals have for staying home and avoiding public places.

The decision of individuals to engage in public avoidance behavior during an epidemic need not be a straightforward one. On the one hand, avoiding public gatherings reduces the likelihood of contacts with infected individuals—and hence the chances of acquiring an infection. On the other hand, staying home means forgoing certain activities – such as going to work or school, shopping, vacationing, or socializing – that contribute to the well-being of an individual. Therefore, choosing whether to engage in public avoidance when there is an outbreak entails evaluating the relative cost and benefit of staying home and reducing contact with other people.

As pointed out in a recent paper on this topic (Chen et al., 2011), the result of this cost-benefit analysis for an individual regarding the merit of public avoidance depends critically on the actions of other people—specifically, the decisions of other people to engage in public avoidance themselves. There are two reasons for this: (i) other people's decision to stay home or go out in public affects the rate with which an individual out in public will come into contact with others; and (ii) other people's decision to stay home or got out can – if there is a significant difference in how much public interactions infected individuals and uninfected individuals engage in – affect the proportion of infected people who are out in public. Both factors influence a susceptible individual's probability of getting infected if the individual chooses to go out in public. Therefore, other people's actions affect an individual's cost of going out in public, and hence an individual's decision to avoid public places.

In the parlance of game theory, there is *strategic interaction* between the actions of individuals with regards to public avoidance behavior, and one major objective of this paper is to apply the concepts and tools from game theory to study the incentives that susceptible individuals have for engaging in public avoidance behavior during an epidemic. It is important to point out that this is a key distinction between the earlier paper on this topic (Chen et al., 2011) and the current paper. The earlier paper assumes that the players in the public avoidance game ('game' in the sense of game theory) have adaptive expectations regarding the behavior of other players: players use the past behavior of others to

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forecast their future behavior; more specifically, the behavior of players at any time period is a best response to the actions of other players in the previous time period. By contrast, in the current paper, the solution concept of a Nash equilibrium from game theory is employed to study the behavior of players in the public avoidance game: in any time period, the behavior of every player is a best response to the actions of other players in the same time period. As we will see, the results of the analysis depend critically on the assumptions that we make regarding players' expectations of how other players will behave in the public avoidance game.

A second objective of the paper here is to compare the equilibrium of the public avoidance game – the outcome that obtains when all players act out of self-interest – with the social optimum—the outcome that is best for the community as a whole. It is well known from the voluntary vaccinations literature that, in general, the equilibrium amount of vaccinations is too low relative to the socially optimal amount (Fine and Clarkson, 1986; Brito et al., 1991). This obtains since there is a positive externality associated with vaccinations: one's act of getting immunized confers a benefit on other people, but these external benefits do not factor into a self-interested individual's vaccination decision; hence there is a divergence between individual interest and collective interest. In particular, since the benefit to an individual of getting vaccinated is lower than the collective benefit of having that individual vaccinated, there is too little – relative to the socially optimal outcome – vaccination in equilibrium. We will see, in the analysis that follows, whether or not these insights and conclusions gleaned from the study of voluntary vaccinations carry over to the case of public avoidance behavior. Specifically, the paper considers how the equilibrium amount of public avoidance compares to that in the socially optimal outcome.

Although public avoidance behavior during epidemics is also analyzed in a recent paper by Reluga (2010), it should be noted that in his model, an individual's likelihood of getting infected depends only on the individual's own action and is completely unaffected by the actions of other people. This means, in particular, that in his analysis an individual's number of contacts, as well as the fraction of those contacts that are infected, are assumed not to depend on the behavior of other people. Therefore, the strategic interactions between the actions of individuals with regards to public avoidance behavior – one major focus of the paper here – are entirely absent in Reluga's model. In fact, as we will see later, the Reluga model can be considered a special case of the general model presented in this paper.

The analysis here reveals that the set of Nash equilibria of the public avoidance game, as well as how the equilibria compare to the social optimum, depend on the contact function that governs the rate at which encounters occur in public, specifically on how the rate of contact varies with the number of people who choose to go out in public. If the *contact ratio* – defined here to be the ratio of the contact rate to the number of people out in public – is increasing with the number of people out in public, then there exists a unique Nash equilibrium. Moreover, in equilibrium, the amount of public avoidance is too low with respect to social welfare. On the other hand, if the contact ratio is decreasing in the number of people out in public, then there can be multiple Nash equilibria, none of which is in general socially optimal. Furthermore, the amount of public avoidance in equilibrium with a decreasing contact ratio is too high in that social welfare can be increased if more susceptible individuals choose to go out in public. In the special case where the contact ratio does not vary with the number of people out in public, there is a unique Nash equilibrium, and it is also the socially optimal outcome.

The paper is organized as follows. Section 2 introduces the model. The set of Nash equilibria of the public avoidance game is characterized in Section 3. In Section 4, the socially optimal outcome of the public avoidance game is considered, and comparisons between the set of equilibria and the social optimum are made. A summary of the results, as well as concluding remarks, are given in Section 5.

## 2. The model

The model is a variant of the earlier one presented in Chen et al. (2011). Consider a susceptible-infected-recovered (SIR) model in discrete time with a continuum of agents. At each point in time, an agent can be in one of three health states: susceptible, infected, and recovered. An infected agent recovers at the end of any period with probability $\rho \in [0,1]$. A recovered agent is fully immune and can never be infected. Assume that there is no entry or exit of agents so that the population size is constant over time. For convenience, let us normalize the size of the population to be of measure 1.

The behavior of agents is specified as follows. In every period, agents choose how much time $\alpha \geq 0$ to spend outside the home in public during that period. This variable $\alpha$ will henceforth be referred to as the *level of public activity*. Individuals' choices of their public activity levels affect the rate at which contacts occur in the population and, hence, the rate with which an infectious disease spreads. Assume that $\alpha$ belongs to the interval $[0,\overline{\alpha}]$, where $\overline{\alpha}$ can be interpreted as agents' "normal" public activity level in the absence of any infectious diseases. Without loss of generality, let $\overline{\alpha}=1$. The level of public avoidance in the model is thus captured by $1-\overline{\alpha}$.

As is standard in the economic epidemiology literature, it is assumed that agents are self-interested and seek to maximize their own payoff without regard to the payoffs of other agents. In addition, assume that, all else being equal, an agent prefers less public avoidance over more. Since recovered agents cannot be infected, they have no incentive to engage in public avoidance behavior; thus recovered agents always choose $\alpha=1$ for their level of public activity.

Similarly, self-interested infected agents would not choose to adopt public avoidance behavior. However, the state of being infected can be sufficiently debilitating to cause some infected agents to have to stay home. Let us assume that, in any period, the fraction $1-\gamma$ of infected agents, where $\gamma \in [0,1]$, are too sick to engage in any public activities. The remaining infected agents – those that have only mild symptoms – fully participate in public activities, i.e., $\alpha=1$ for these infected agents.

Let us now consider the contact structure and the disease transmission process. A susceptible agent who chooses public activity level $\alpha_t$ in time $t$ has probability $\alpha_t\lambda_t$ of being infected in that period, where $\lambda_t$ denotes the probability of infection in period $t$ per unit public activity level. This probability $\lambda_t$ is a function of the disease prevalence $p_t$ as well as the public activity levels chosen by all agents at time $t$. Now, letting $r_t$ denote the time $t$ fraction of recovered agents, the mean level of public activity in the population at time $t$ is $n_t \equiv \gamma p_t + r_t + \alpha_t(1-p_t-r_t)$ if all susceptible agents choose public activity level $\alpha_t$ in period $t$. Suppose $\lambda_t$ is specified as follows:

$$\lambda_t = \lambda(p_t, r_t, \alpha_t) = m(n_t)\frac{\beta\gamma p_t}{n_t},$$

where $\beta \in (0,1]$ is the transmission probability, and $m$ is the *meeting* or *contact function* that specifies the rate at which a susceptible agent encounters other agents per unit of public activity. Note that $\beta\gamma p_t/n_t$ is the probability of acquiring an infection conditional on meeting another agent.